Lecture 2
Op-Amp Building Blocks and Applications

- **Instrumentation Amplifiers**

- **Filters**

- **Integrators**

- **Differentiators**

- **Frequency-Gain Relation**

- **Non-Linear Op-Amp Applications**

- **DC Imperfections**

Check List for Selecting Op-Amps

- Power Supply

- Gain Bandwidth

- Cost

- Voltage Offset

- Stability

- Output Current

- Noise

- Special Functions

Types of Op-Amps

- Precision Amplifiers:
  - Low noise
  - Low Offset Voltage
  - Low Input Bias Current.
- Low Power/Low Bandwidth
- Video Amplifiers
- Audio Amplifiers
- High Voltage/High Current
- Differential/Instrumentation Amplifiers

Instrumentation Amplifier (IA) Applications

- Used in Environments with High Noise.

- Offers high Input Impedance.

- Low Bias Currents.

- Gain Determined by One Resistor.

- Commercial IA is based on Standard 3-OP Amps.

- Requires Several Matched Resistors.

Instrumentation Amplifier
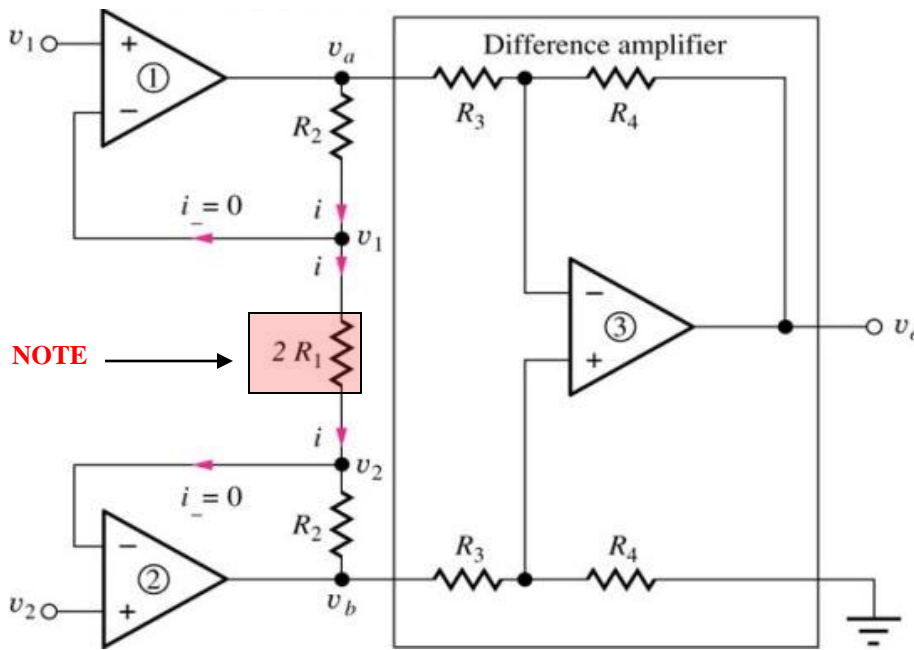
$$v_o = -\frac{R_4}{R_3}(v_a - v_b)$$

$$v_a - iR_2 - i(2R_1) - iR_2 = v_b$$

$$i = \frac{v_1 - v_2}{2R_1}$$

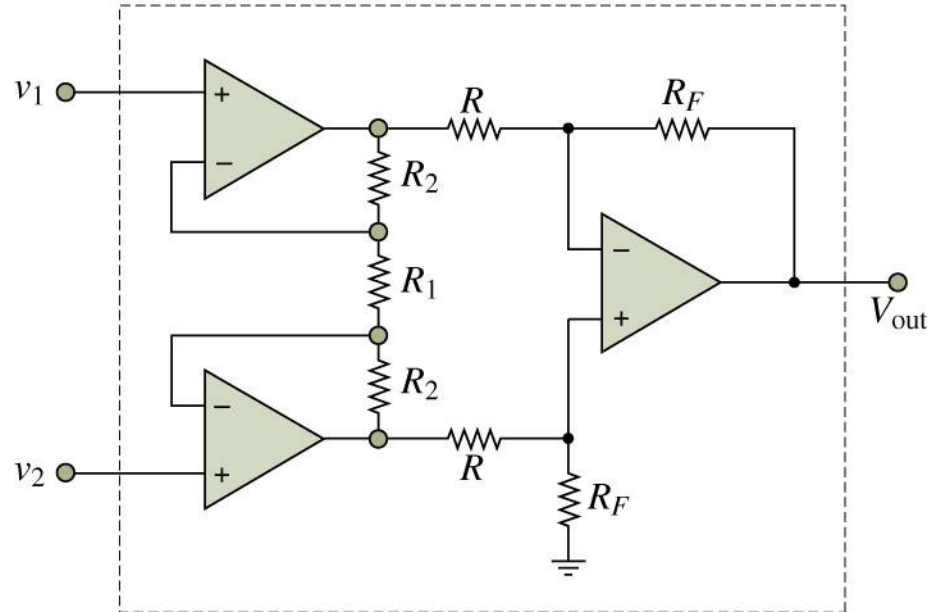$$\therefore v_o = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)(v_1 - v_2)$$

Combines 2 non-inverting amplifiers with the difference amplifier to provide higher gain and higher input resistance.

Ideal input resistance is infinite because input current to both op amps is zero. The CMRR is determined only by Op-Amp 3.
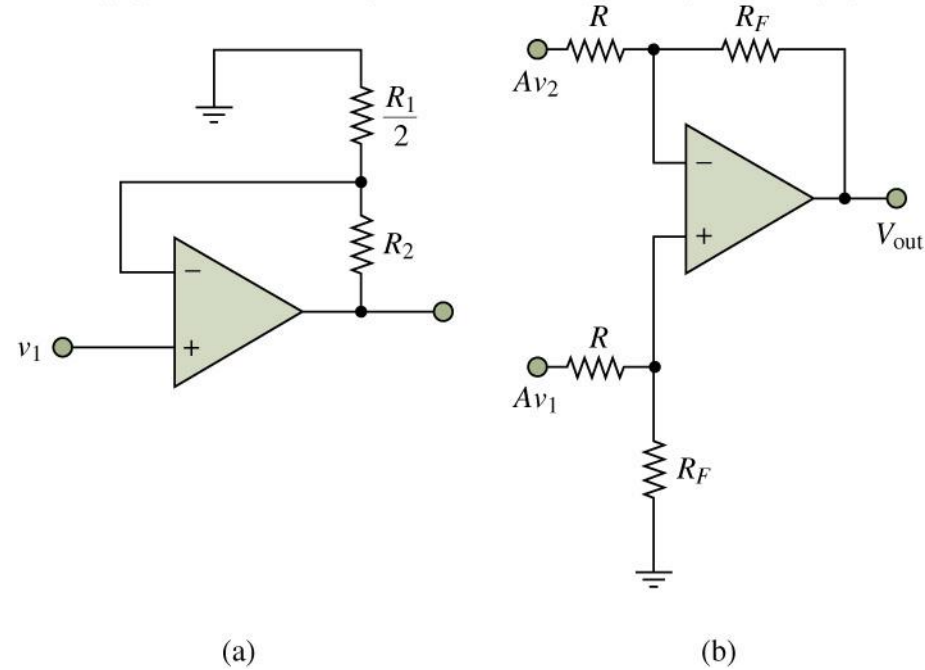
Instrumentation Amplifier

Input (a) output (b) Stages of IA

(a)

(b)

$$A_V = \frac{v_{out}}{v_1 - v_2} = \frac{R_F}{R}\left(1 + \frac{2R_2}{R_1}\right)$$

Instrumentation Amplifier: Example

- **Problem**: Determine V_o

- **Given Data**: $R_1 = 15$ kΩ, $R_2 = 150$ kΩ, $R_3 = 15$ kΩ, $R_4 = 30$ kΩ $V_1 = 2.5$ V, $V_2 = 2.25$ V

- **Assumptions**: Ideal op amp. Hence, $v_- = v_+$ and $i_- = i_+ = 0$.

- **Analysis**: Using dc values,

$$A_{dm} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) = -\frac{30\text{k}\Omega}{15\text{k}\Omega}\left(1 + \frac{150\text{k}\Omega}{15\text{k}\Omega}\right) = -22$$
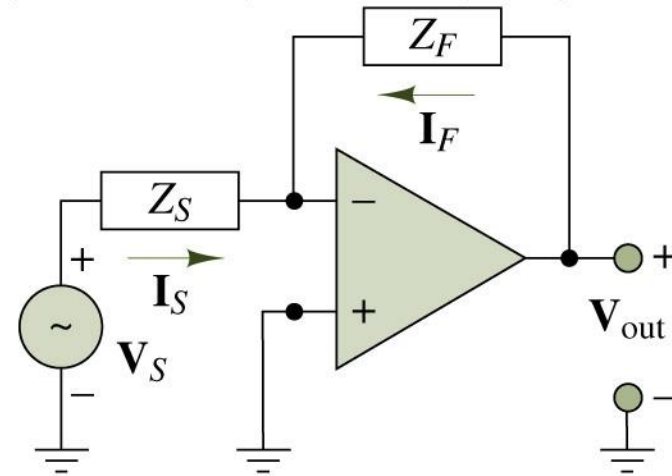
$$V_o = A_{dm}(V_1 - V_2) = -22(2.5 - 2.25) = -5.50\text{V}$$
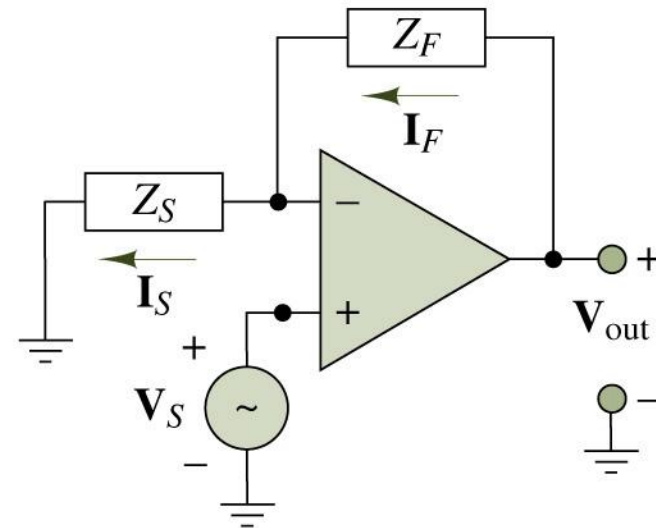
Op-amp Circuits Employing Complex Impedances

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

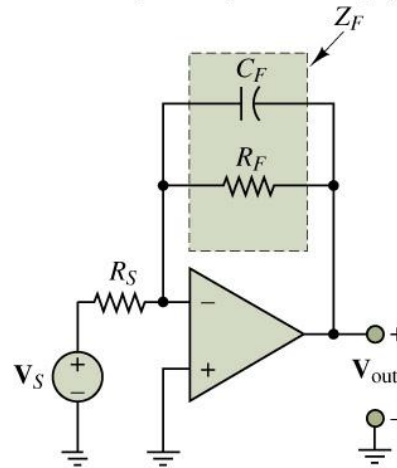$$\frac{V_{out}}{V_S}(j\omega) = 1 + \frac{Z_F}{Z_S}$$

Inverting

Noninverting

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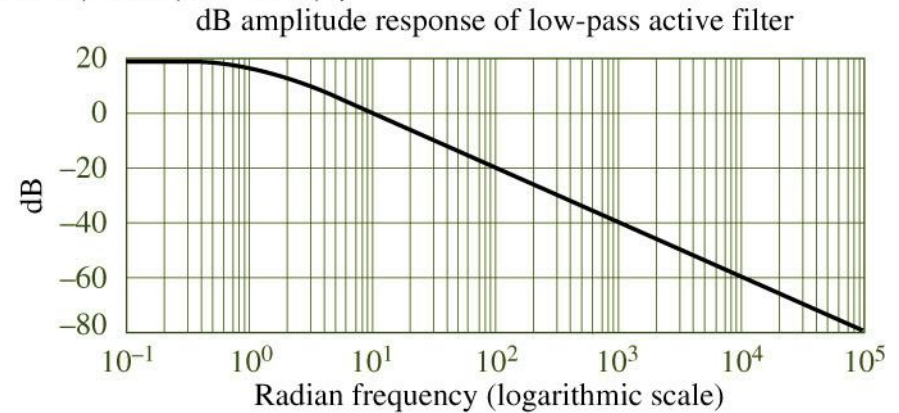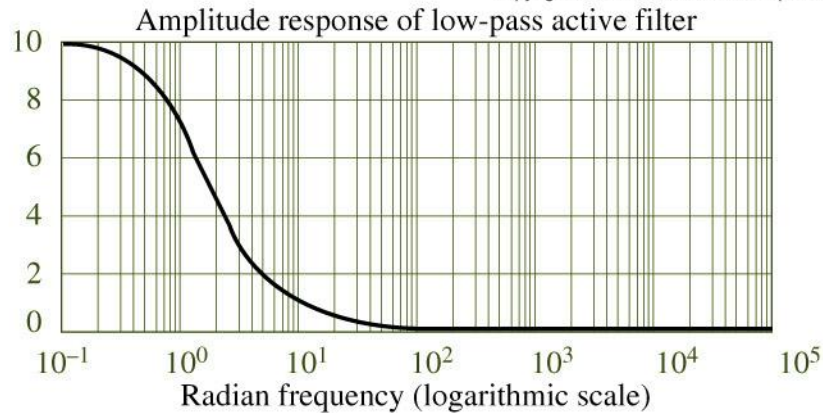# Active Low-Pass Filter

Z_F

C_F

R_F

R_S

V_S $+$ $-$

$-$

$+$

V_{out} $+$

$-$

Normalized Response of Active Low-Pass Filter

Amplitude response of low-pass active filter

Amplitude ratio

10
8
6
4
2
0

10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5

Radian frequency (logarithmic scale)

dB amplitude response of low-pass active filter

dB

20
0
−20
−40
−60
−80

10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5

Radian frequency (logarithmic scale)

The Active Low-Pass Filter

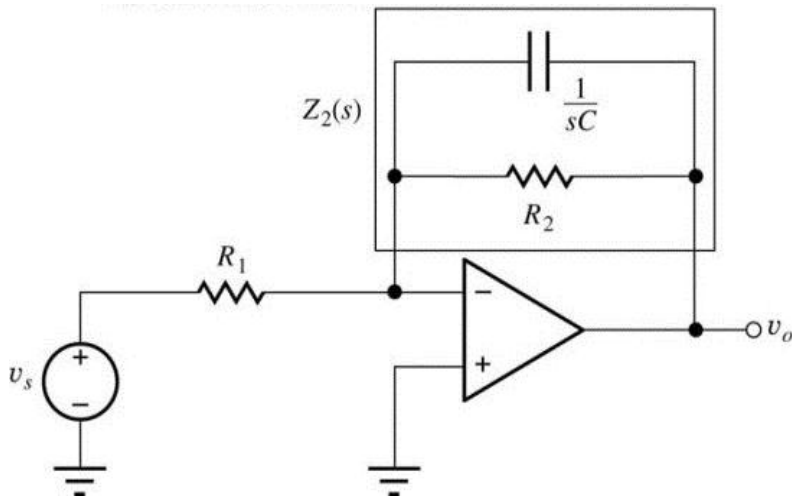Use a phasor approach to gain analysis of this inverting amplifier. Let s = jω.
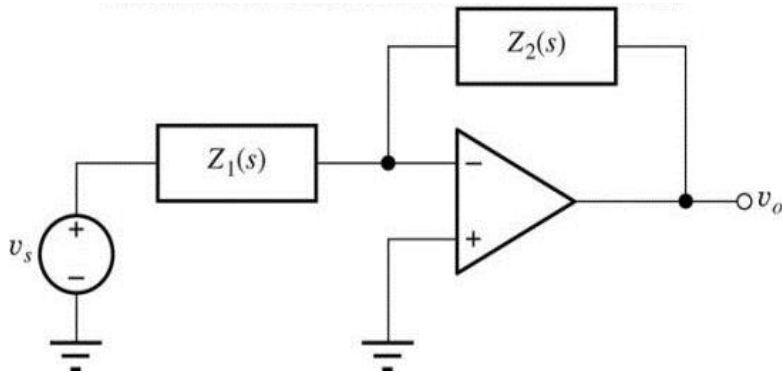
$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)} \qquad Z_1(j\omega) = R_1$$

$$Z_2(j\omega) = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$
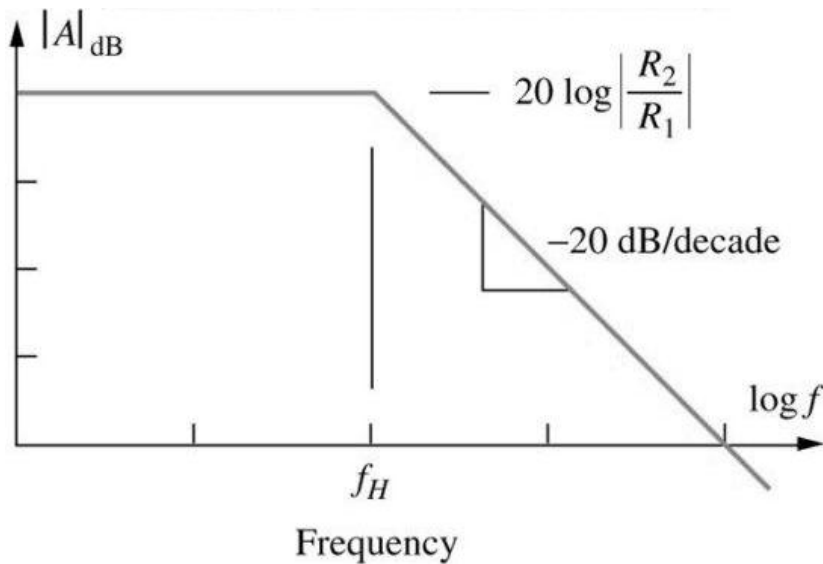
$$A_v = -\frac{R_2}{R_1}\frac{1}{(1+j\omega C R_2)} = \frac{R_2}{R_1}\frac{e^{j\pi}}{(1+\frac{j\omega}{\omega_c})}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_2 C} \qquad \therefore f_c = \frac{1}{2\pi R_2 C}$$

f_c is called the high frequency "cutoff" of the low-pass filter.

Active Low-Pass Filter

- At frequencies below f_c (f_H in the figure), the amplifier is an inverting amplifier with gain set by the ratio of resistors R_2 and R_1.

- At frequencies above f_c, the amplifier response "rolls off" at -20dB/decade.

- Notice that cutoff frequency and gain can be independently set.

$$A_v = \frac{R_2}{R_1}\left|\frac{e^{j\pi}}{(1+\frac{j\omega}{\omega_c})}\right| = \frac{R_2}{R_1\sqrt{1^2+\left(\frac{\omega}{\omega_c}\right)^2}}\left|\frac{e^{j\pi}}{e^{j\tan^{-1}(\omega/\omega_c)}}\right| = \frac{R_2}{R_1\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}e^{j[\pi-\tan^{-1}(\omega/\omega_c)]}$$

magnitude   phase

Active Low-Pass Filter: Example

- **Problem**: Design an active low-pass filter
- **Given Data**: $A_v = 40$ dB, $R_{in} = 5$ kΩ, $f_H = 2$ kHz
- **Assumptions:** Ideal op amp, specified gain represents the desired low-frequency gain.
- **Analysis**: $|A_v| = 10^{40\text{dB}/20\text{dB}} = 100$

  Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1.
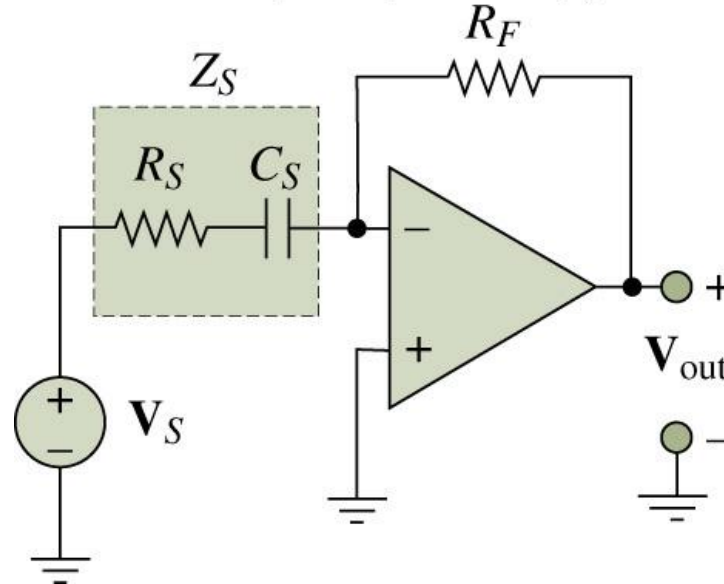
  The cutoff frequency is then set by C.

  $$R_1 = R_{in} = 5\text{k}\Omega \qquad \text{and} \qquad |A_v| = \frac{R_2}{R_1} \Rightarrow R_2 = 100 R_1 = 500\text{k}\Omega$$

  $$C = \frac{1}{2\pi f_H R_2} = \frac{1}{2\pi (2\text{kHz})(500\text{k}\Omega)} = 159\text{pF}$$

  The closest standard capacitor value of 160 pF lowers cutoff frequency to 1.99 kHz.
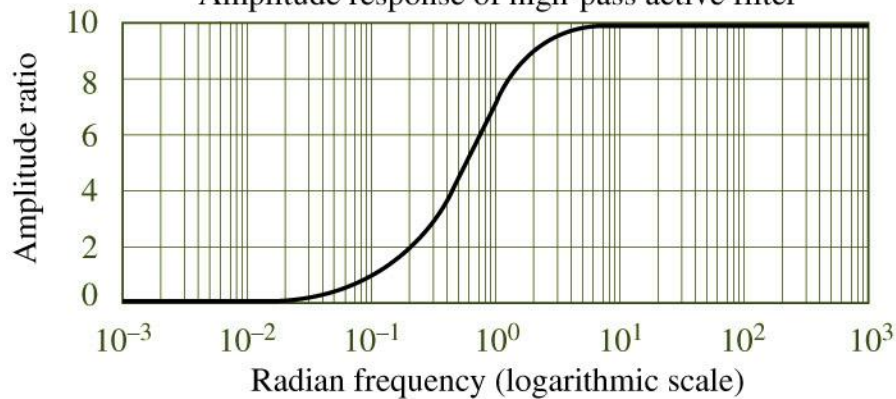
Active High-Pass Filter

Normalized Response of Active High-Pass Filter

Amplitude response of high-pass active filter

dB amplitude response of high-pass active filter

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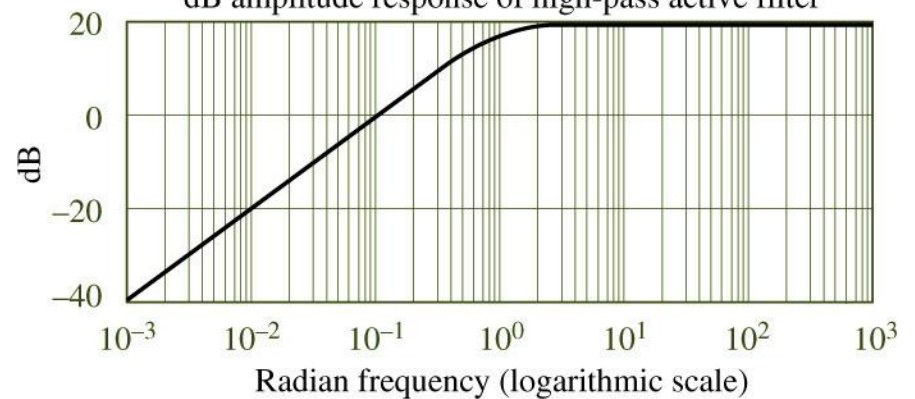
Active Band-Pass Filter
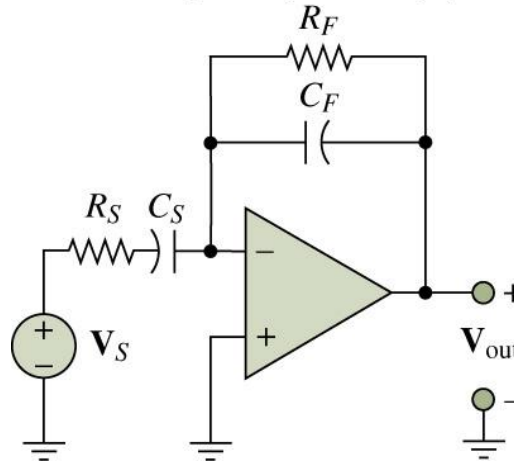
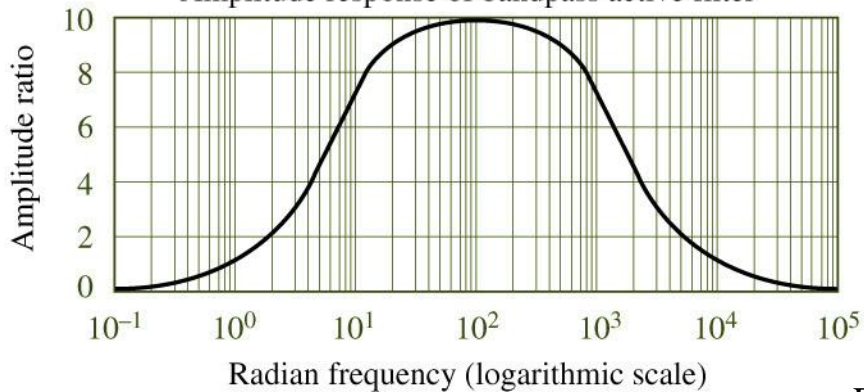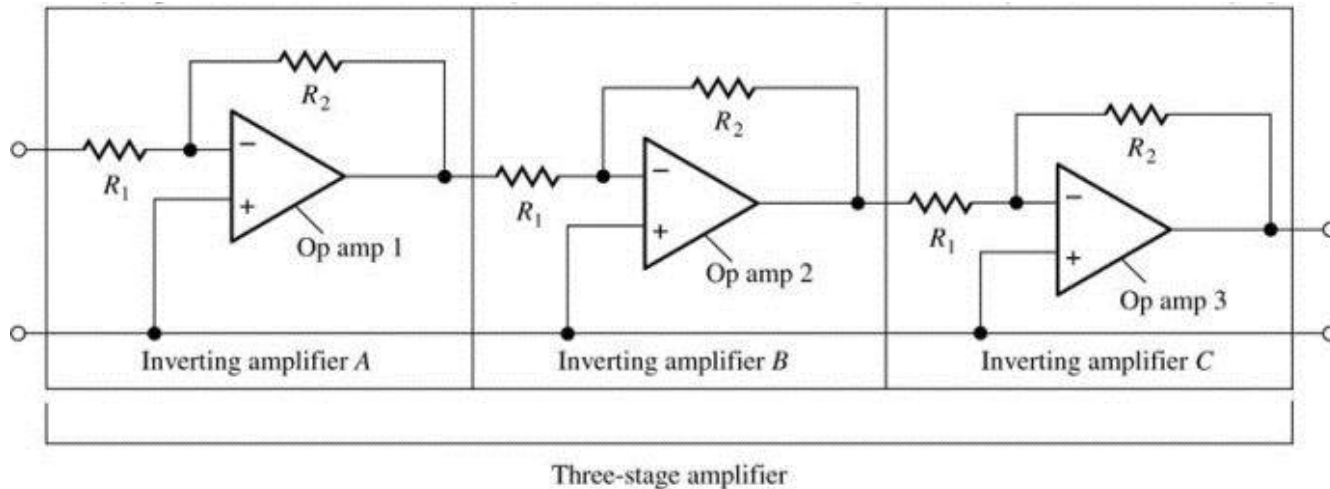## Normalized Amplitude Response of Active Band-Pass Filter

Amplitude response of bandpass active filter

dB amplitude response of bandpass active filter
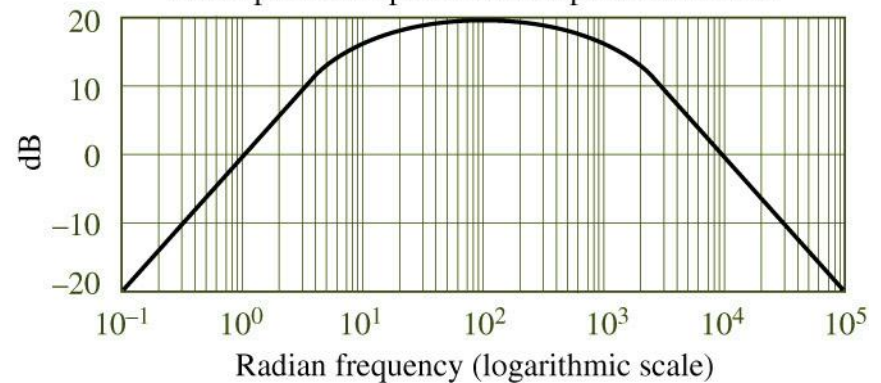
Cascaded Amplifiers

Three-stage amplifier

- Connecting several amplifiers in cascade (output of one stage connected to the input of the next) can meet design specifications not met by a single amplifier.

- Each amplifer stage is built using an op amp with parameters A, R_{id}, R_o, called open loop parameters, that describe the op amp with no external elements.

- A_v, R_{in}, R_{out} are closed loop parameters that can be used to describe each closed-loop op amp stage with its feedback network, as well as the overall composite (cascaded) amplifier.

Two-Port Model for a 3-Stage Cascade Amplifier
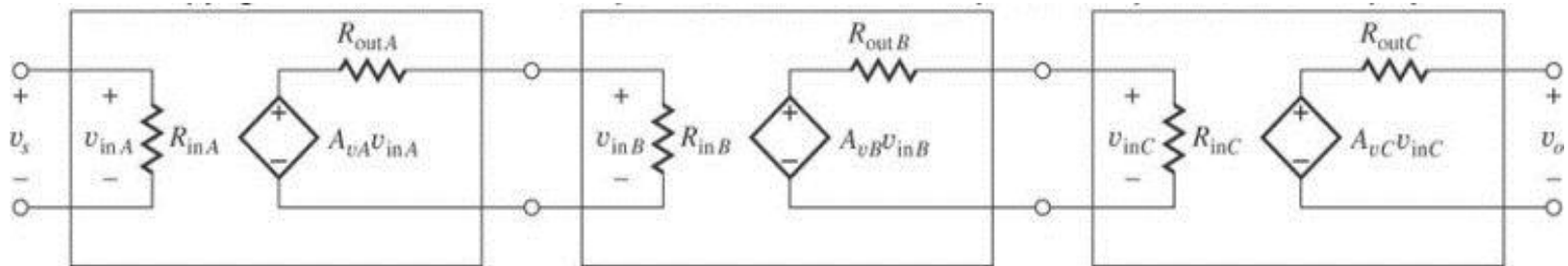
- Each amplifier in the 3-stage cascaded amplifier is replaced by its 2-port model.

$$v_o = A_{vA} v_s \left(\frac{R_{inB}}{R_{outA} + R_{inB}} \right) A_{vB} \left(\frac{R_{inC}}{R_{outB} + R_{inC}} \right) A_{vC}$$

Since $R_{out} = 0$
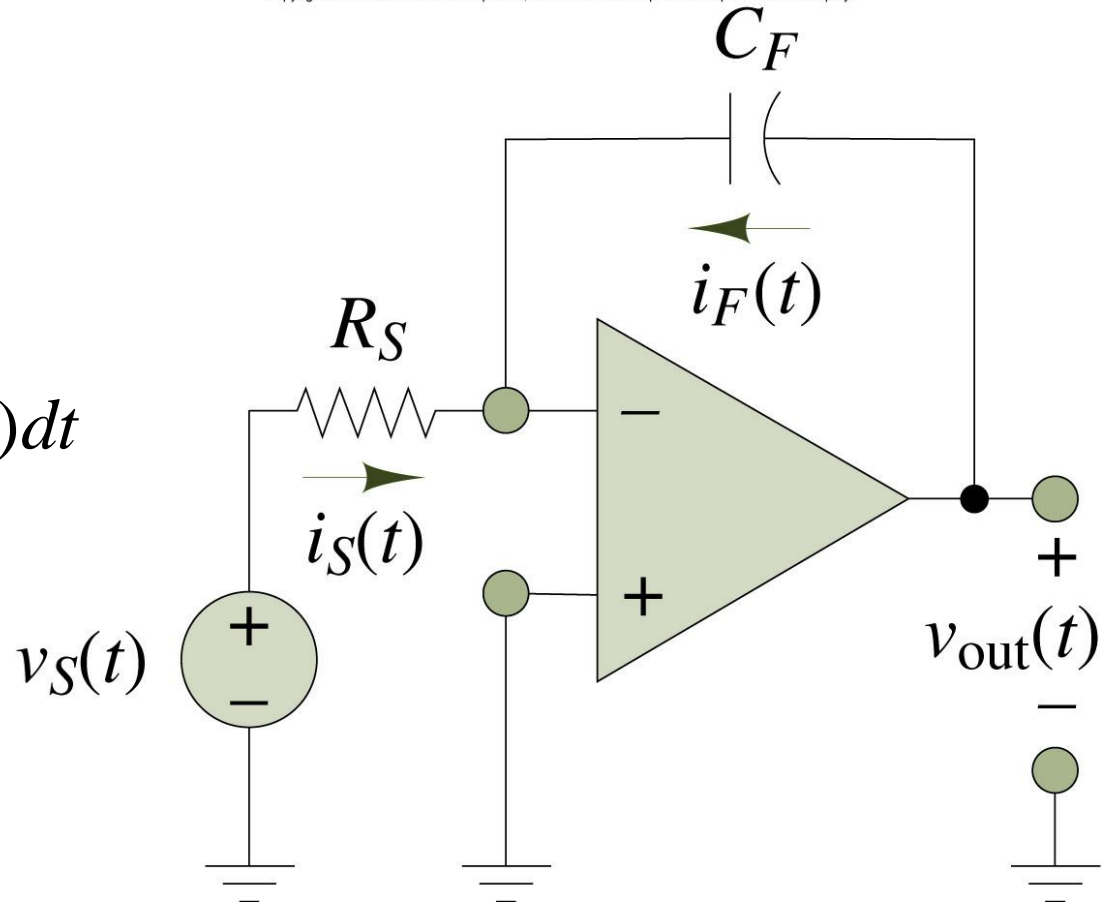
$$A_v = \frac{v_o}{v_s} = A_{vA} A_{vB} A_{vC}$$

$$R_{in} = R_{inA} \quad \text{and} \quad R_{out} = R_{outC} = 0$$

Op-Amp Integrator

$$v_{out}(t) = -\frac{1}{R_S C_F} \int_{-\infty}^{t} v_S(t)\,dt$$

Inverting Integrator

Now replace resistors R_a and R_f by complex
components Z_a and Z_f, respectively, therefore

Supposing $\quad V_o = \dfrac{-Z_f}{Z_a} V_{in}$

(i)   The feedback component is a capacitor C,
i.e.,

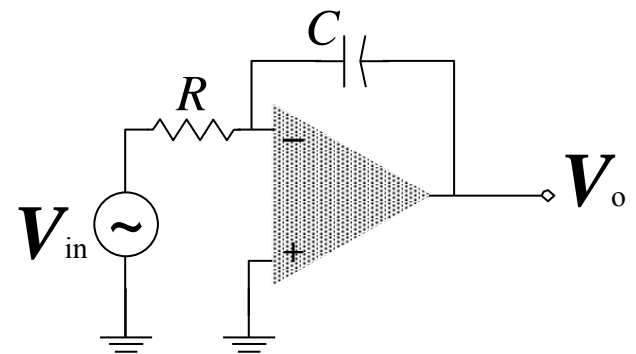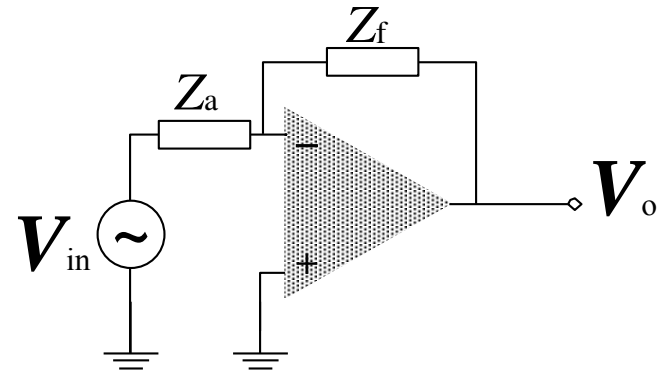$$Z_f = \frac{1}{j\omega C}$$

(ii)   The input component is a resistor R, $Z_a = R$
Therefore, the closed-loop gain (V_o/V_{in}) become:

$$v_o(t) = \frac{-1}{RC} \int v_i(t)\,dt$$

where $\quad v_i(t) = V_i e^{j\omega t}$

What happens if $Z_a = 1/j\omega C$ whereas, $Z_f = R$?
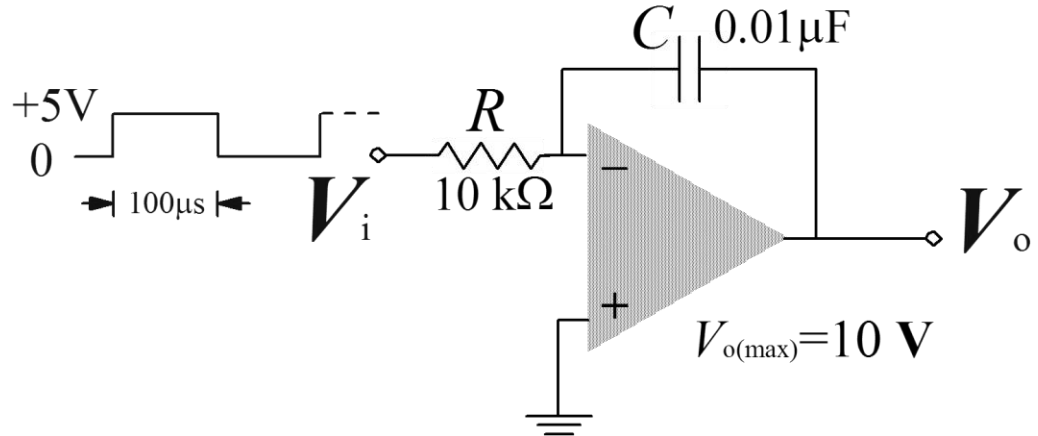Inverting differentiator

Op-Amp Integrator

Example:

(a) Determine the rate of change of the output voltage.
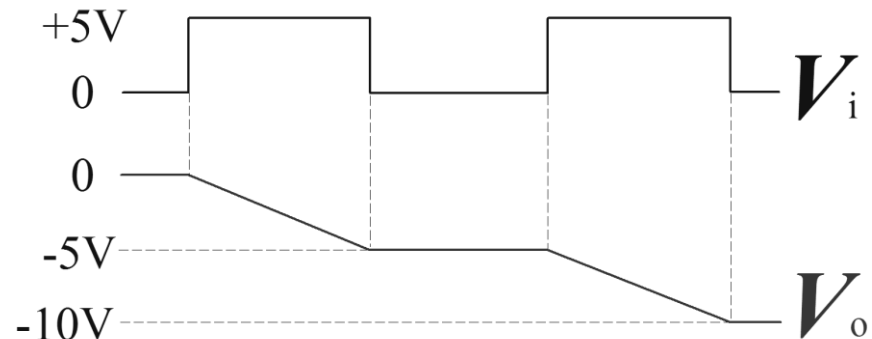
(b) Draw the output waveform.

Solution:

(a) Rate of change of the output voltage

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_i}{RC} = \frac{5\,\mathbf{V}}{(10\,\mathbf{k\Omega})(0.01\,\mu\mathbf{F})}$$

$$= -50\,\mathbf{mV/}\mu\mathbf{s}$$

(b) In 100 μs, the voltage decrease

$$\Delta V_o = (-50\,\mathbf{mV/}\mu\mathbf{s})(100\mathbf{\mu s}) = -5\mathbf{V}$$

Op-Amp Differentiator

$$v_{out}(t) = -R_F C_S \frac{dv_S(t)}{dt}$$

R_F

$i_F(t)$

C_S

$-$

$+$

$i_S(t)$

$v_S(t)$

$v_{out}(t)$

R

C

0

t_0 t_1 t_2 $\boldsymbol{V}_i$

$\boldsymbol{V}_o$

0
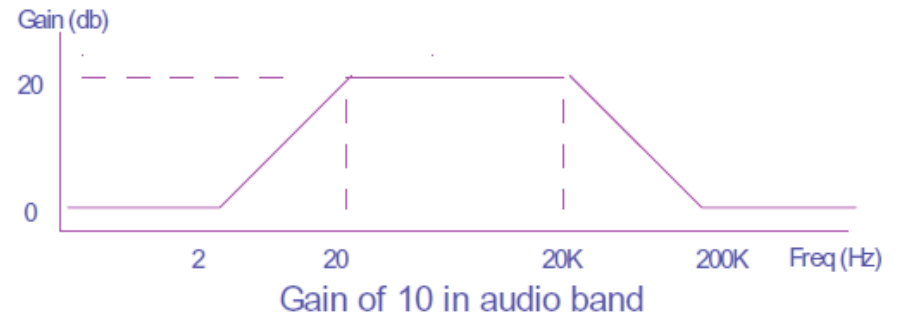
t_0 t_1 t_2

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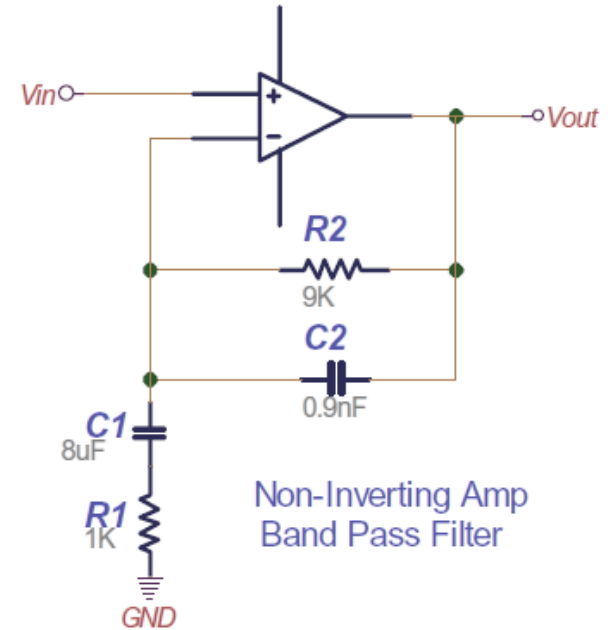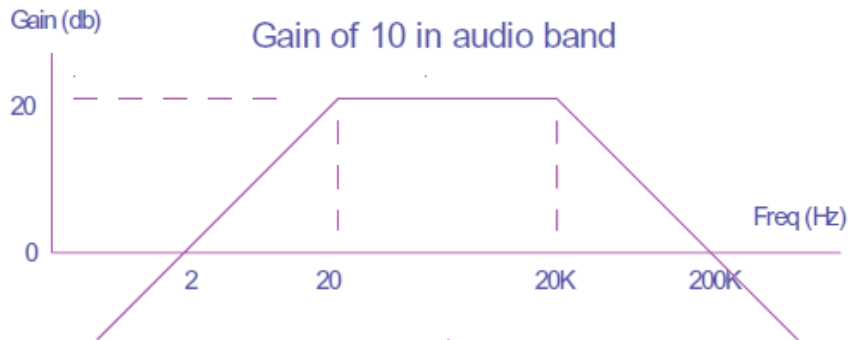
20
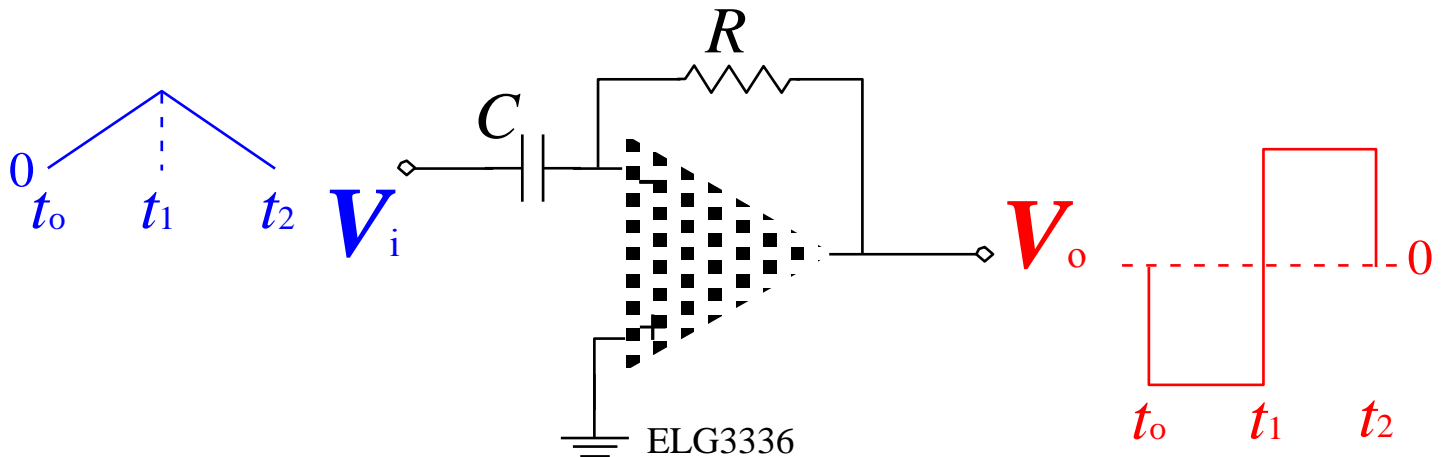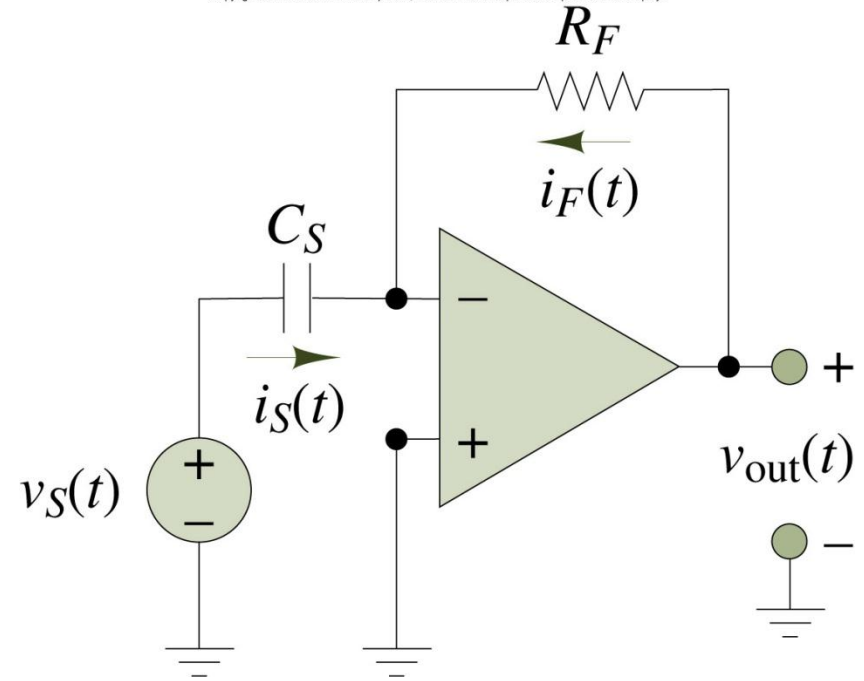
Frequency-Gain Relation

- Ideally, signals are amplified from DC to the highest AC frequency
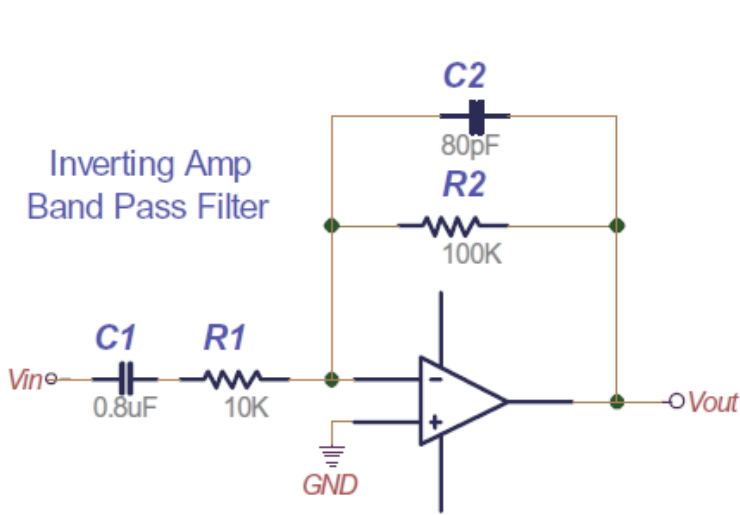
- Practically, bandwidth is limited

- 741 family op-amp have an limit bandwidth of few KHz.

- Unity Gain frequency f_1: the gain at unity

- Cutoff frequency f_c: the gain drop by 3dB from dc gain G_d

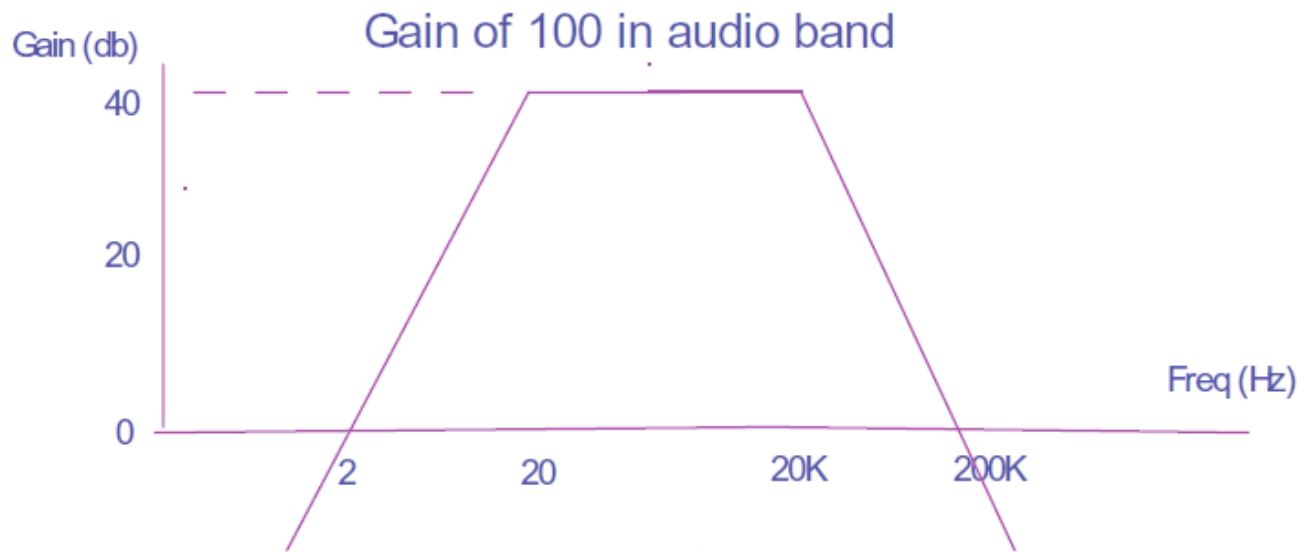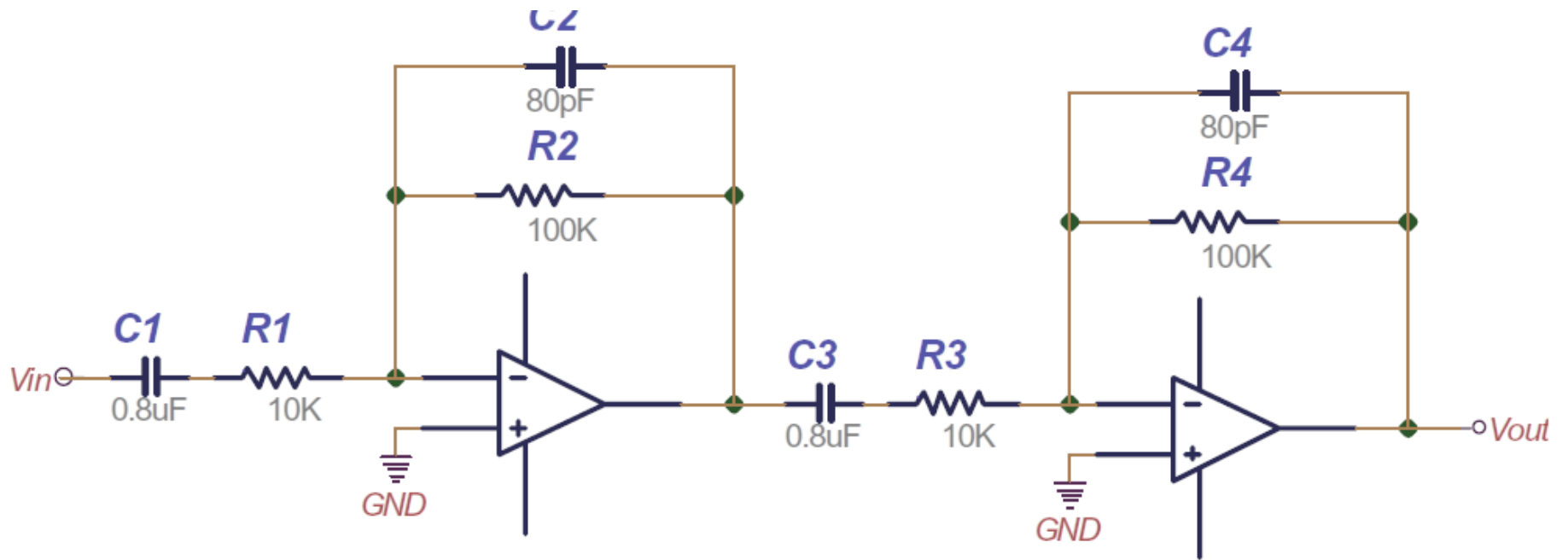$$\boxed{\text{GB Product}: f_1 = G_d f_c}$$

(Voltage Gain)

$20\log(0.707) = 3\text{dB}$

G_d

$0.707 G_d$

1

0

f_c

f_1
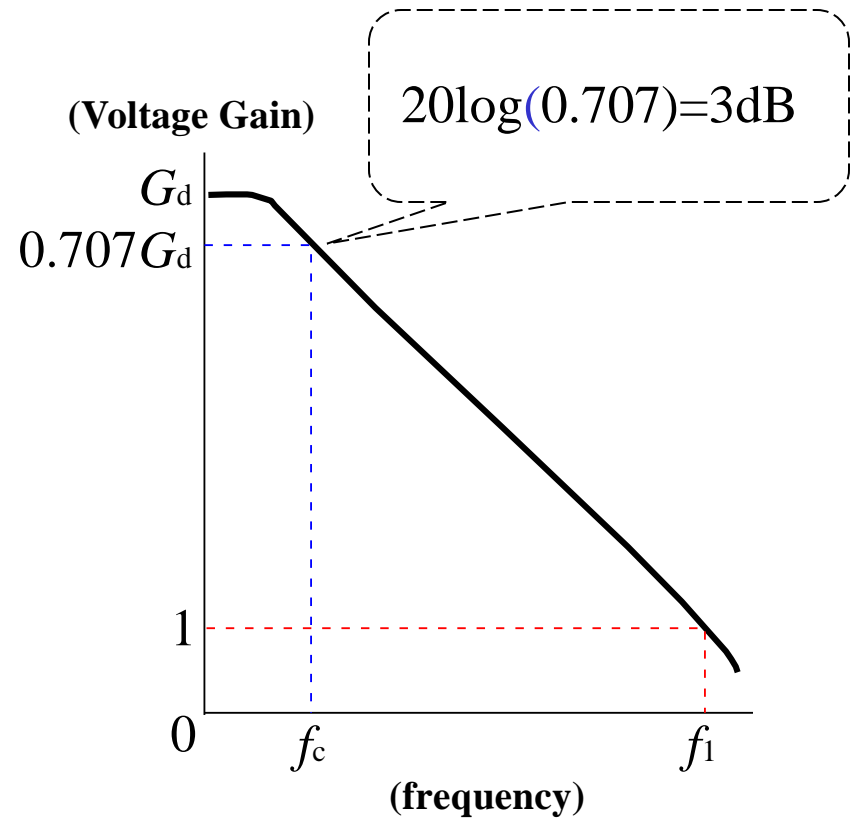
(frequency)

Building Filters: Gain and Bandwidth

Gain Bandwidth Product = Gain x Bandwidth

Gain of 100 in audio band

Gain-Bandwidth Product

Example: Determine the cutoff frequency of an op-amp having a unit gain frequency f_1 = 10 MHz and voltage differential gain G_d = 20**V**/m**V**
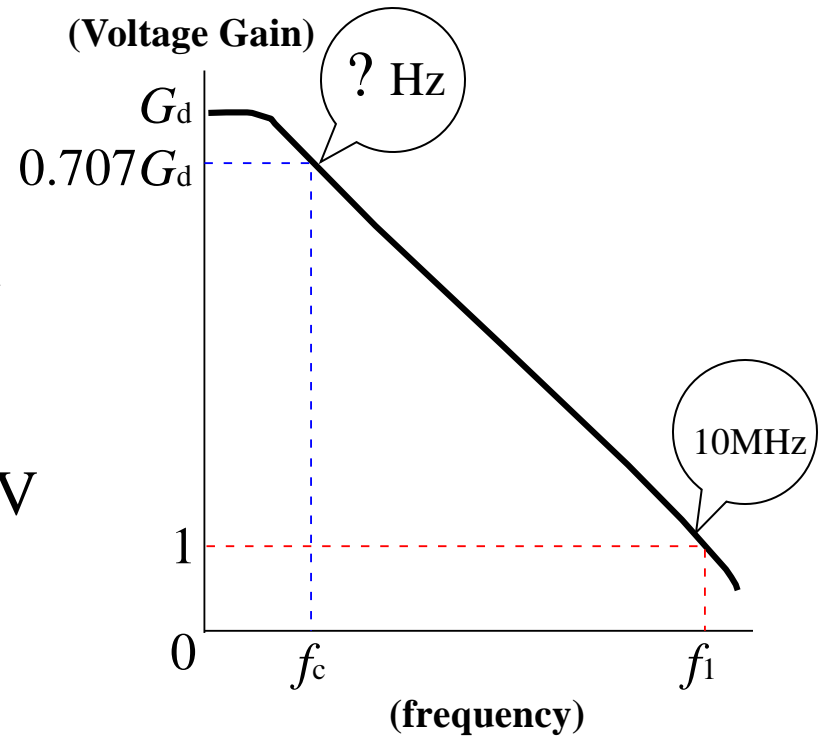
Sol:

Since f_1 = 10 MHz

By using GB production equation

$$f_1 = G_d f_c$$

$$f_c = f_1 / G_d = 10 \text{ MHz} / 20 \text{ V/mV}$$
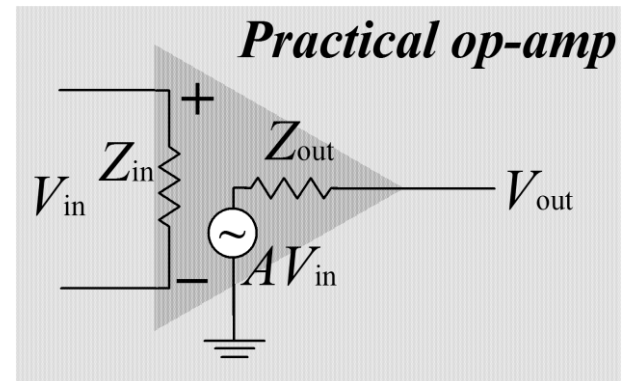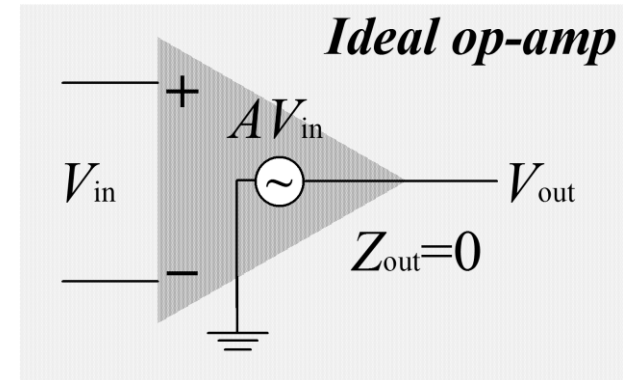
$$= 10 \times 10^6 / 20 \times 10^3$$

$$= 500 \text{ Hz}$$

Ideal Versus Practical Op-Amp

	Ideal	**Actual**
Open Loop gain A	∞	10^5
Bandwidth BW	∞	10-100Hz
Input Impedance Z_{in}	∞	$>1M\Omega$
Output Impedance Z_{out}	$0\ \Omega$	10-100 Ω
Output Voltage V_{out}	Depends only on $V_d = (V_+ - V_-)$ Differential mode signal	Depends slightly on average input $V_c = (V_+ + V_-)/2$ Common-Mode signal
CMRR	∞	10-100dB

Ideal op-amp

V_{in} AV_{in} V_{out} $Z_{out}=0$

Practical op-amp
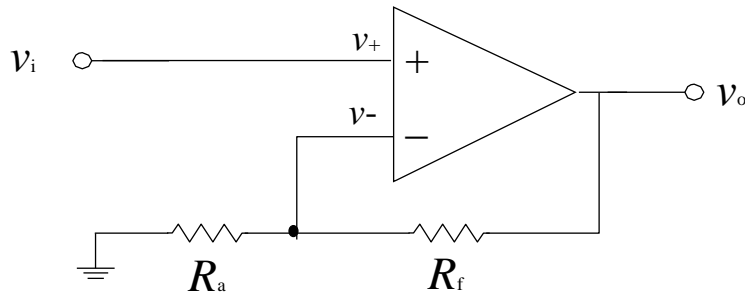
V_{in} Z_{in} Z_{out} V_{out} AV_{in}

Analysis

Ideal Op-Amp Main Properties:

(1)   The voltage between V_+ and V_- is zero $V_+ = V_-$

(2)   The current into both V_+ and V_- termainals is zero
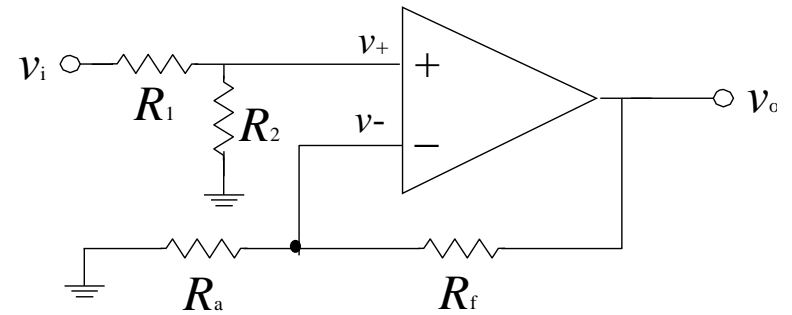
For ideal Op-Amp circuit:

(1)   Write the kirchhoff node equation at the noninverting terminal V_+

(2)   Write the kirchhoff node eqaution at the inverting terminal V_-

(3)   Set $V_+ = V_-$ and solve for the desired closed-loop gain

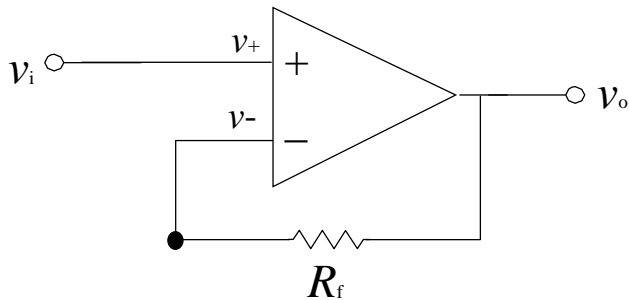Noninverting amplifier

$$v_o = (1 + \frac{R_f}{R_a})v_i$$

Noninverting input with voltage divider

$$v_o = (1 + \frac{R_f}{R_a})(\frac{R_2}{R_1 + R_2})v_i$$

Voltage follower

$$v_o = v_i$$
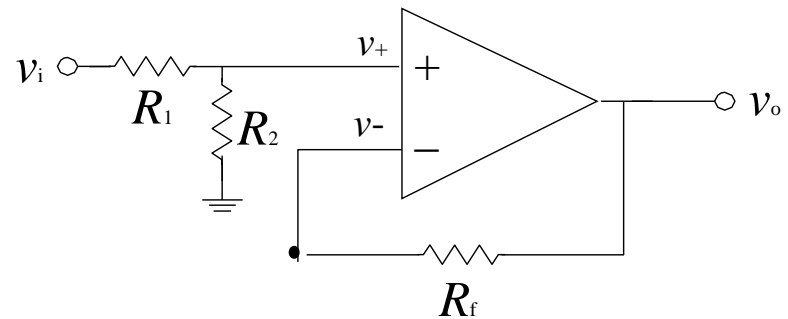
Less than unity gain

$$v_o = \frac{R_2}{R_1 + R_2}v_i$$

Non-Linear Op-Amp Applications

- Applications using saturation
  - Comparators
  - Comparator with hysteresis (Schmitt Trigger)
  - Oscillators.

- Applications using active feedback components
  - Log, antilog, squaring etc. amplifiers
  - Precision rectifier

Comparators

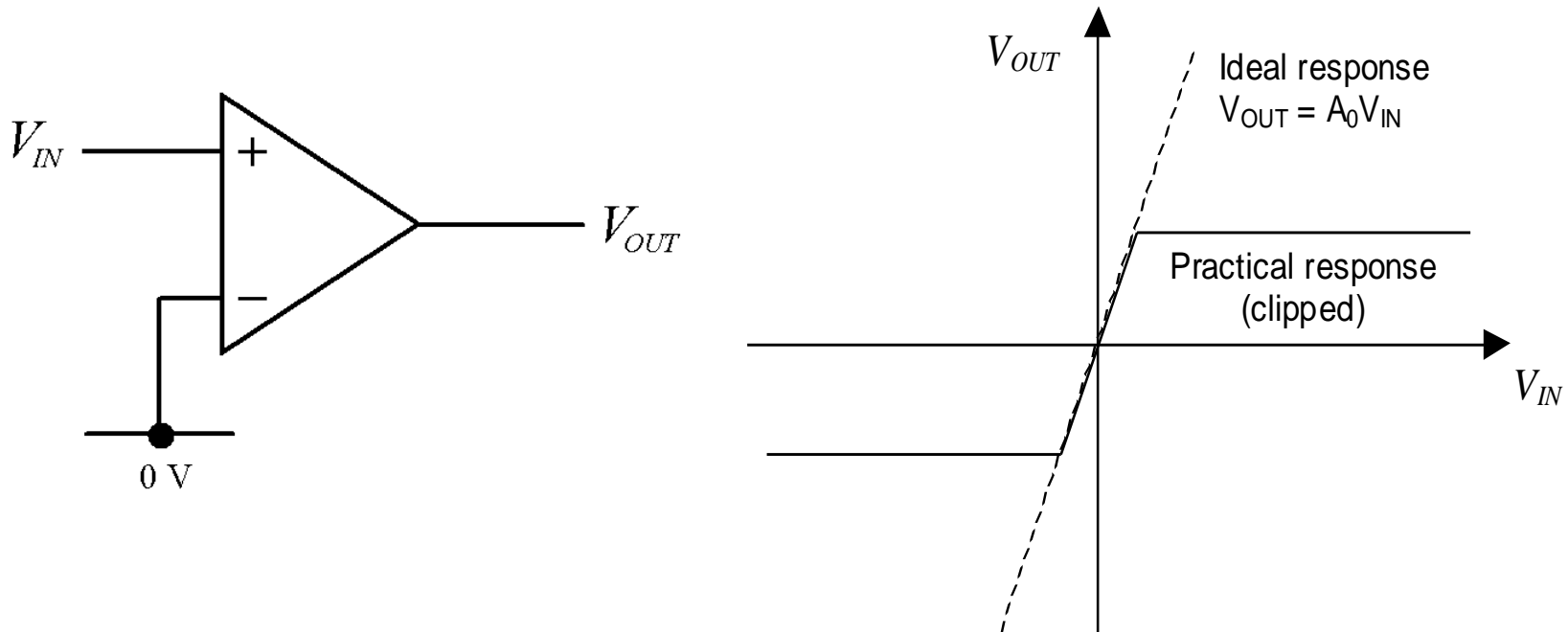A comparator is a device that compares the magnitude of two inputs and gives an output that indicates which of the two is larger!
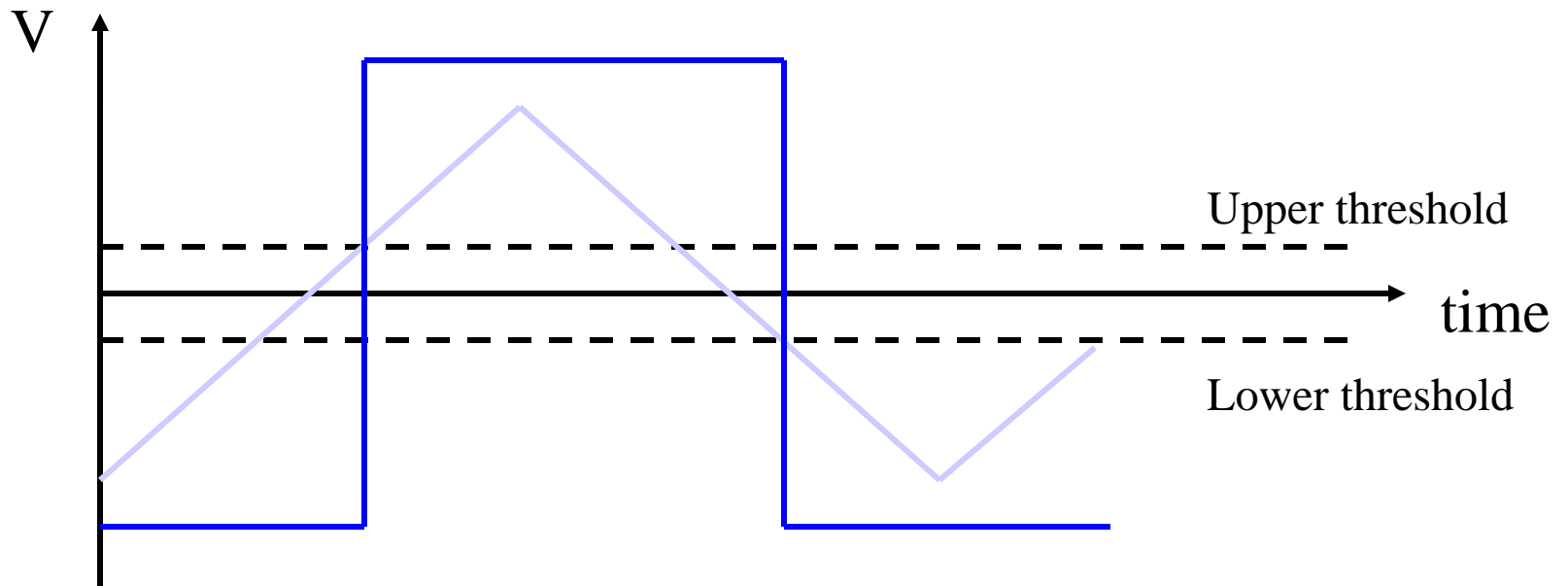

If A_0 is large, practical response can be approximated as :

$$V_{IN} > 0 \Rightarrow V_+ > V_- \Rightarrow V_{OUT} = +V_{SAT}$$
$$V_{IN} < 0 \Rightarrow V_+ < V_- \Rightarrow V_{OUT} = -V_{SAT}$$

Hysteresis

- A comparator with hysteresis has a 'safety margin'.
- One of two thresholds is used depending on the current output state.

Schmitt Trigger

- The Schmitt trigger is an op-amp comparator circuit featuring hysteresis.

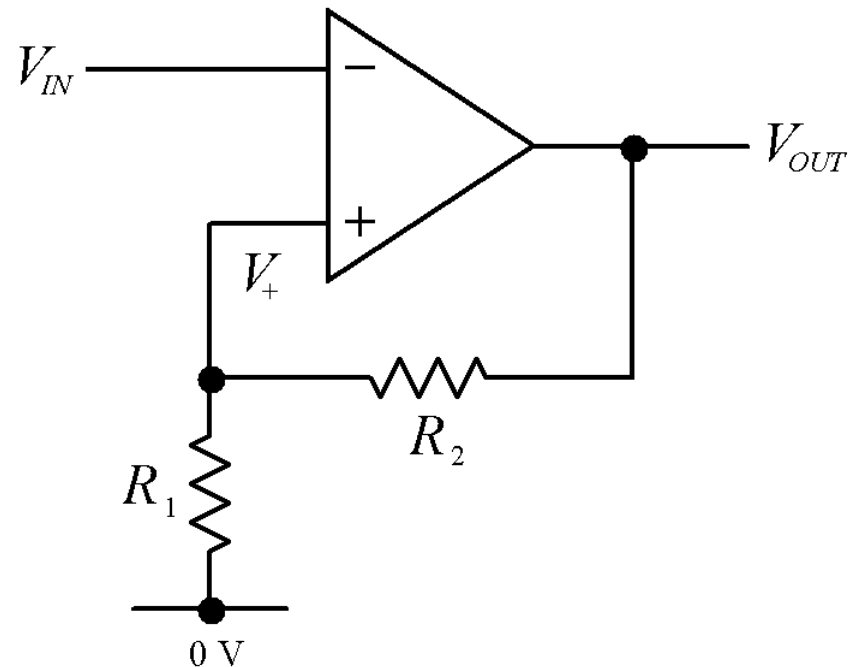- The inverting variety is the most commonly used.

Switching occurs when:

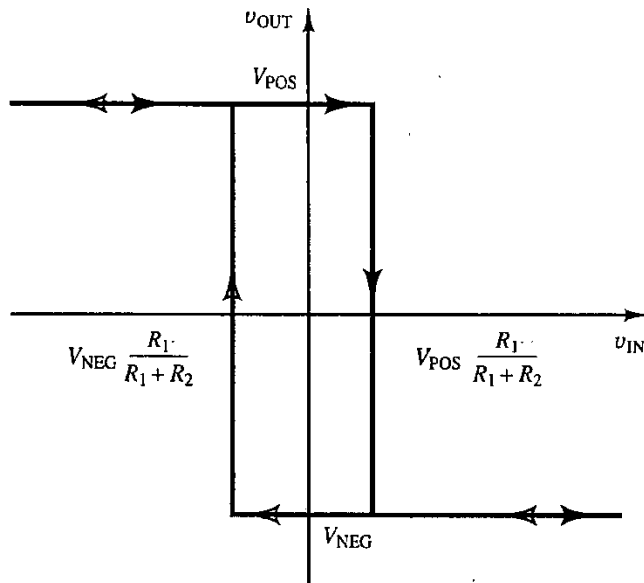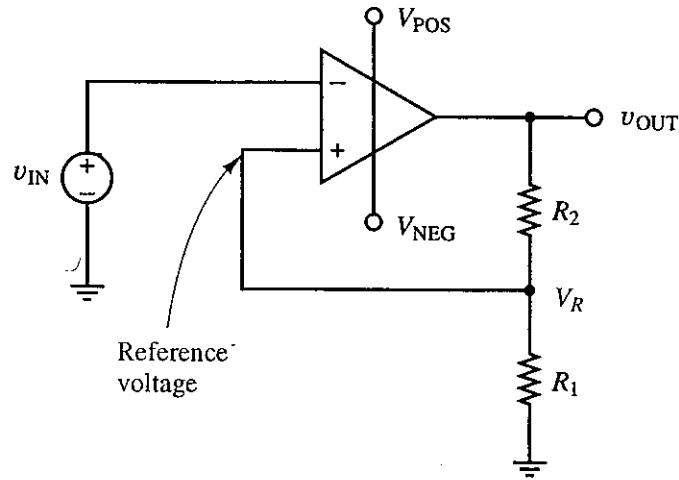$$V_{IN} = V_- = V_+ = V_{OUT} \frac{R_1}{R_1 + R_2}$$

But,

$$V_{OUT} = \pm V_{SAT}$$

$$\therefore V_{THRESH} = \pm V_{SAT} \frac{R_1}{R_1 + R_2}$$

Schmitt Trigger Op-amp Circuit

- The open-loop comparator from the previous two slides is very susceptible to noise on the input
  - Noise may cause it to jump erratically from + rail to − rail voltages
- The Schmitt Trigger circuit (see at the left) solves this problem by using positive feedback
  - It is a comparator circuit in which the reference voltage is derived from a divided fraction of the output voltage, and fed back as positive feedback.
  - The output is forced to either V_{POS} or V_{NEG} when the input exceeds the magnitude of the reference voltage
  - The circuit will remember its state even if the input comes back to zero (has memory)
- The transfer characteristic of the Schmitt Trigger is shown at the left
  - Note that the circuit functions as an inverter with hysteresis
  - Switches from + to − rail when $v_{IN} > V_{POS}(R1/(R1 + R2))$
  - Switches from − to + rail when $v_{IN} < V_{NEG}(R1/(R1 + R2))$

Asymmetrical Thresholds

- We do not need always want the threshold levels to be symmetrical around 0 V.

- More general configuration features an arbitrary reference level.
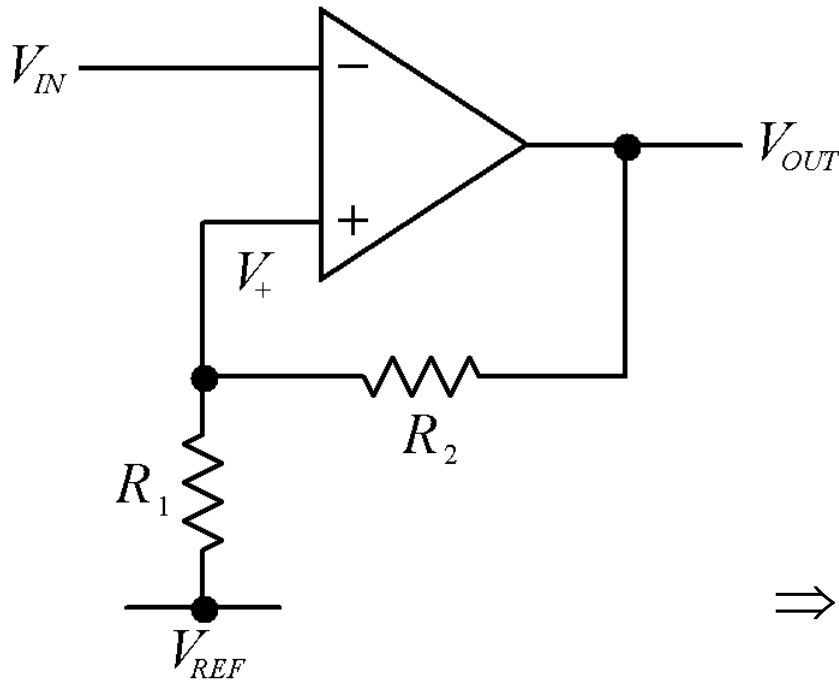
Using Kirchoff's current law:

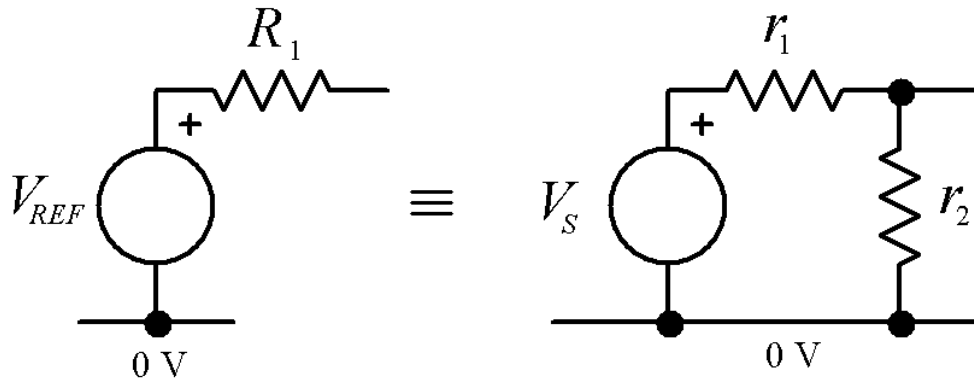$$\frac{V_{OUT} - V_+}{R_2} + \frac{V_{REF} - V_+}{R_1} = 0$$

$$\Rightarrow \frac{V_{OUT}}{R_2} + \frac{V_{REF}}{R_1} = \frac{V_+}{R_2} + \frac{V_+}{R_1} = V_+ \frac{R_1 + R_2}{R_1 R_2}$$

$$\Rightarrow V_+ = V_{OUT} \frac{R_1}{R_1 + R_2} + V_{REF} \frac{R_2}{R_1 + R_2}$$

Realising V$_{REF}$

$$V_{THRESH} = \pm V_{SAT}\,\frac{R_1}{R_1 + R_2} + V_{REF}\,\frac{R_2}{R_1 + R_2}$$

But,

$$\text{Providing} \quad R_1 = r_1 \parallel r_2 \quad \text{and} \quad V_{REF} = V_S\,\frac{r_2}{r_1 + r_2}$$

DC Imperfections

Three DC Imperfections of Real Op-Amps
Input Bias Current; Input Offset Current; and Input Offset Voltage (output voltage may not be zero for zero input voltage)
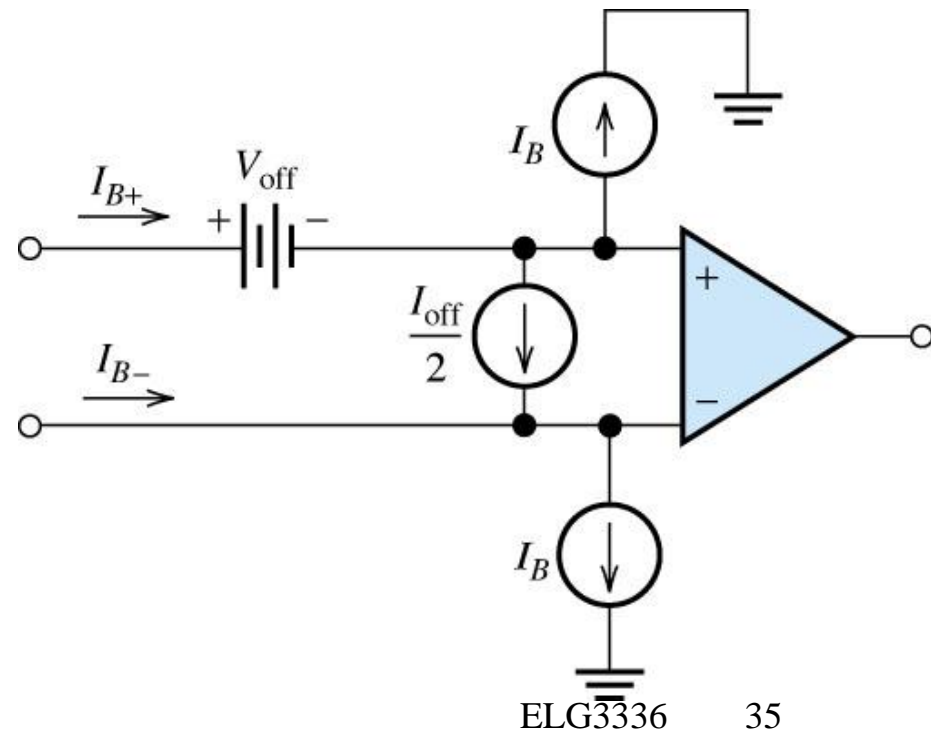
Bias Current: All op-amps draw a small constant DC bias currents at their inputs. Typical value for a 741 is around 100 nA. This is only notable when very high impedance sources are used. In such cases, an alternative op-amp with lower bias current should be used.

Bias Current

$$I_B = \frac{I_{B+} + I_{B-}}{2}$$

Offset Current

$$I_{off} = I_{B+} - I_{B-}$$

Offset Voltage

- When both input voltages are equal, the output should be zero. Actually it probably won't be due to an offset voltage between the inputs. Typically, this is around 2 mV.

- Offset voltage is automatically compensated by a negative feedback network. It can be a problem for precision comparator applications.

- Both the offset voltage and bias current are DC. A.C. operation is not affected by them (they just add an offset) Negative feedback reduces the effect of both. Steps can be taken to reduce them (further reading)

Example: Find the worst-Case DC Output Voltage of an Inverting Amplifier assuming $v_{in} = 0$. The maximum bias current of the Op-Amp is 100 nA. The maximum offset current is 40 nA, and the maximum offset voltage is 2 mV.
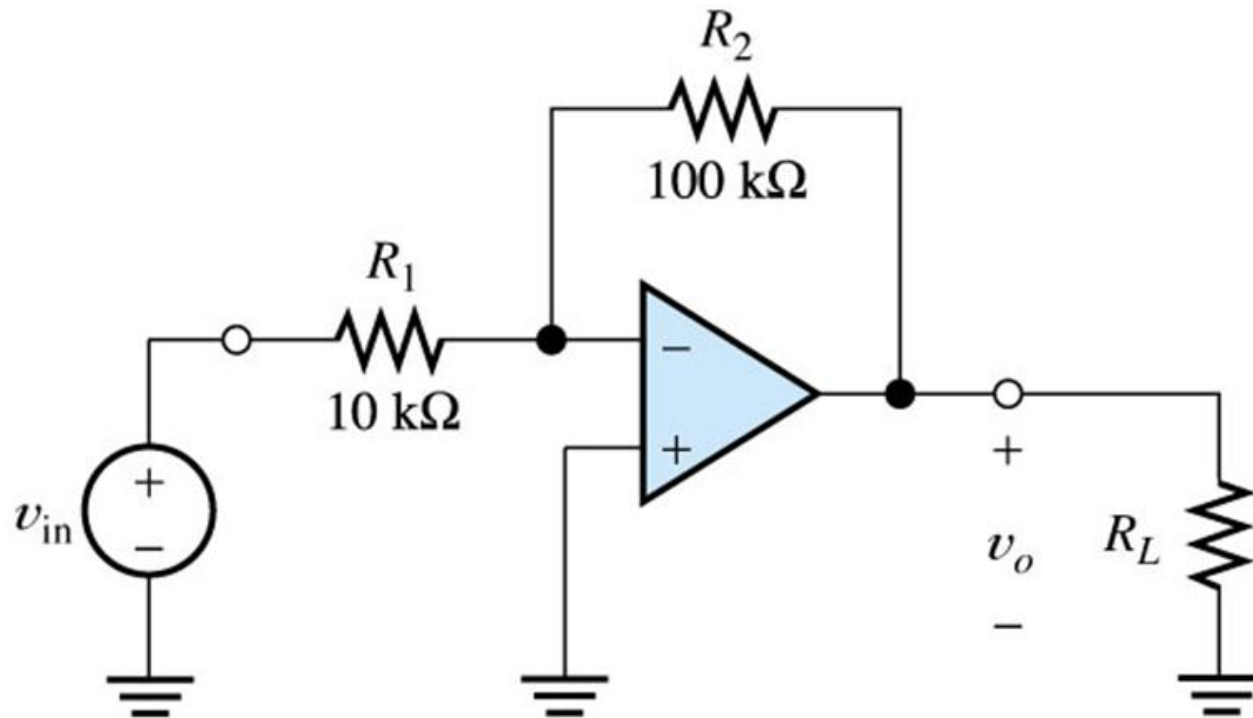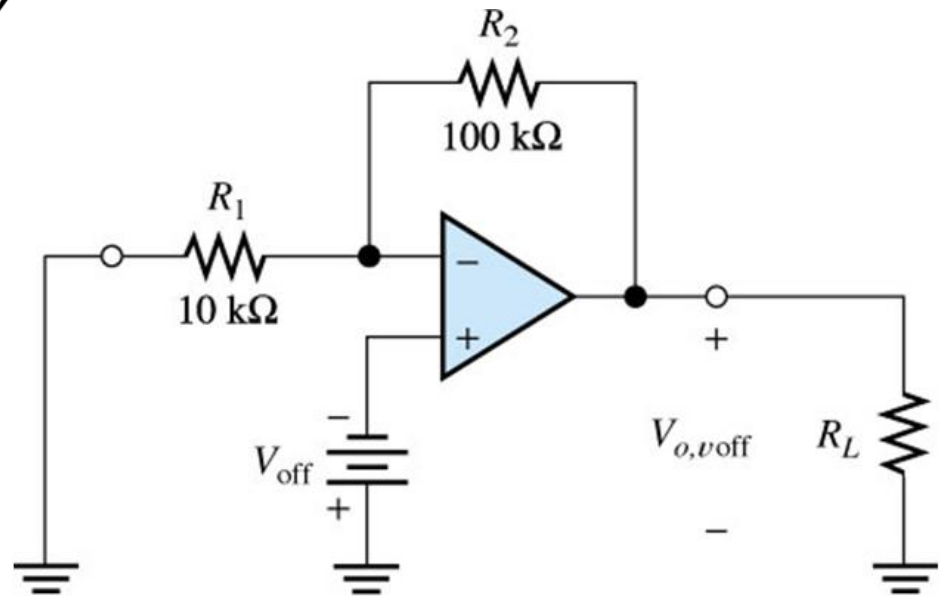

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First, Consider the Offset Voltage

$$V_{0,voff} = -\left(1 + \frac{R_2}{R_1}\right) V_{off}$$

$$V_{0,off} = -11 V_{off}$$
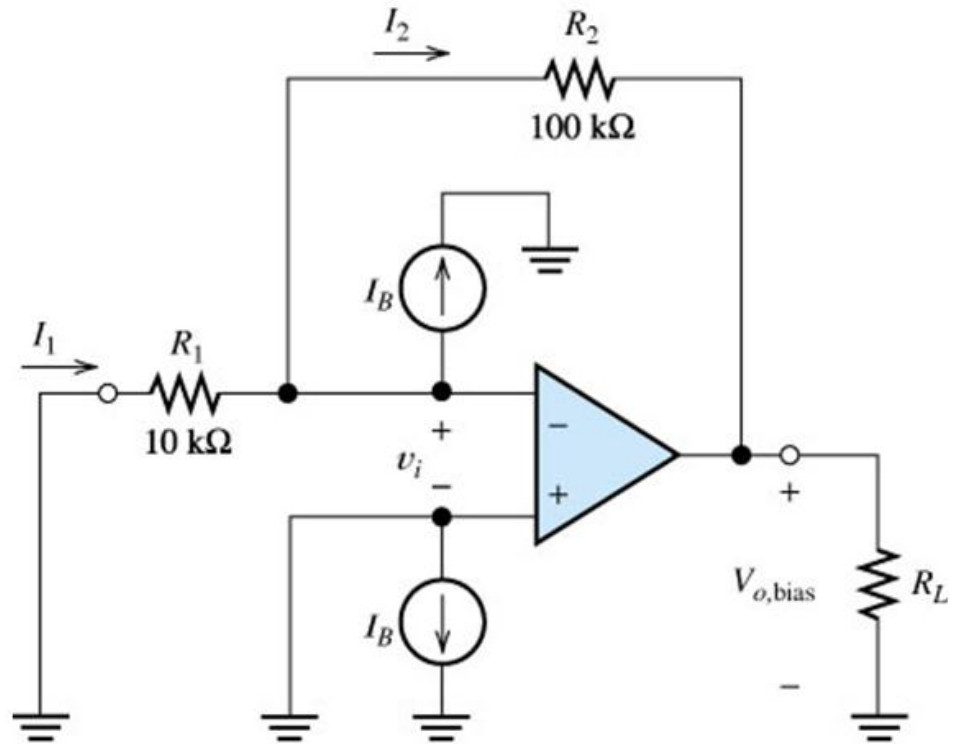

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$$V_{0,off} = -22 \text{ and } + 22mV$$

Second, Bias Current Sources

- $V_{0,bias} = -R_2 I_2 - R_1 I_1$

- $I_1 = 0$
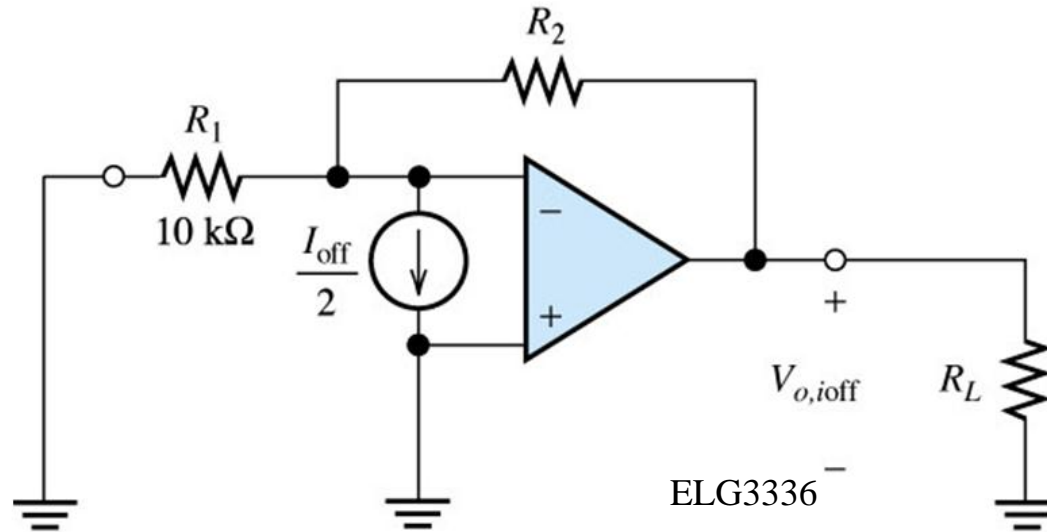
- $I_2 = -I_B$

- $V_{0,bias} = R_2 I_B$

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Third, Offset Current Source

- $V_{0,ioff} = R_2 \left(\frac{I_{off}}{2}\right) = $ -2 and 2 mV

- $V_0 = V_{0,voff} + V_{o,bias} + V_{o,ioff}$

- $V_0 = 22 + 10 + 2 = 34\text{mV}$

- $V_0 = -22 + 0 - 2 = \text{-24mV}$