ELG 4125: ELECTRICAL POWER TRANSMISSION AND DISTRIBUTION:

TUTORIAL 3: -

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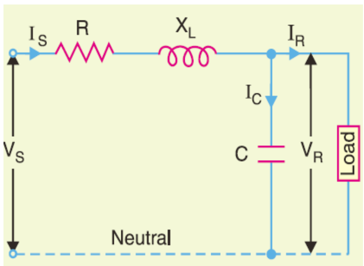
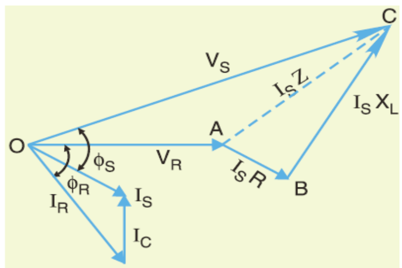
End Condenser method:-





$$\vec{I_S} = \vec{I_R} + \vec{I_C}$$
$$= I_R(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C V_R$$
$$= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi f C V_R)$$

Voltage drop/phase $= \vec{I_S}\vec{Z} = \vec{I_S}(R + jX_L)$

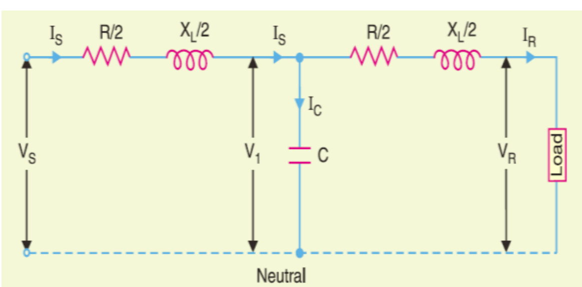
Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\vec{Z} = \vec{V_R} + \vec{I_S}(R + jX_L)$

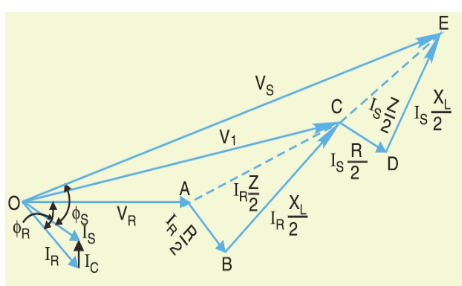
Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$
$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_S^2 R} \times 100$$

Nominal T method:-





Voltage across C, $\quad \vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

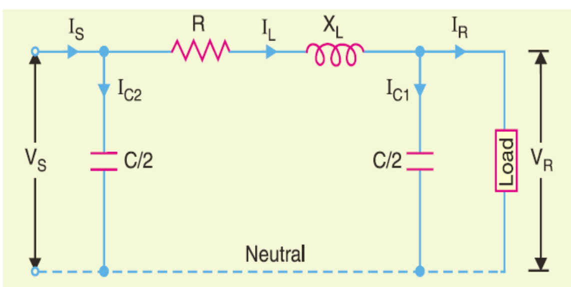
$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

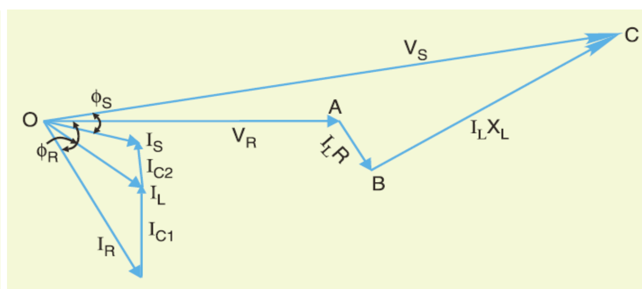
Capacitive current, $\quad \vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\quad \vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\quad \vec{V_S} = \vec{V_1} + \vec{I_S}\,\dfrac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\dfrac{R}{2} + j\dfrac{X_L}{2}\right)$

Nominal π Method:-





$\vec{V_R} = V_R + j\,0$

Load current, $\quad \vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Charging current at load end is

$$\vec{I_{C1}} = j\,\omega\,(C/2)\,\vec{V_R} = j\,\pi\,f\,C\,\vec{V_R}$$

Line current, $\quad \vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

Sending end voltage, $\quad \vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + jX_L)$

Charging current at the sending end is

$$\vec{I_{C2}} = j\,\omega\,(C/2)\,\vec{V_S} = j\,\pi\,f\,C\,\vec{V_S}$$

∴ Sending end current, $\quad \vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

<u>*Examples:-*</u>

1) *A (medium) single phase transmission line 100 km long has the following constants:*

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Ans)

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000$ V

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0 \cdot 8} = 284$ A

$\cos \phi_R = 0{\cdot}8$; $\sin \phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor [see Fig. 10.10 (*ii*)], we have,

$\vec{V_R} = V_R + j\,0 = 66{,}000 V$

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R) = 284\ (0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

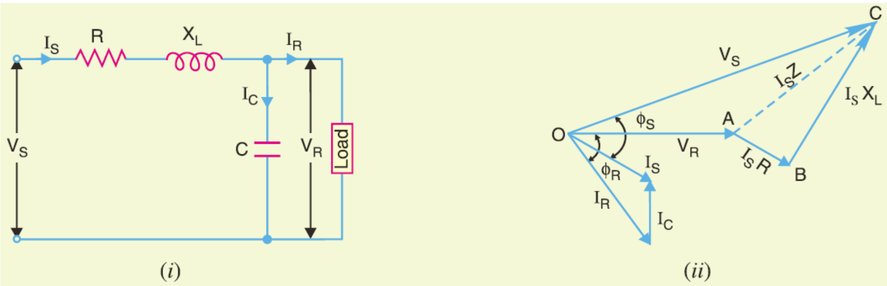


Fig. 10.10

Capacitive current, $\vec{I_C} = j\,Y \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\,(25 + j\,80)$

$= 5{,}675 + j\,18{,}160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_R}$ is :

$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96°$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$\theta_2 = \tan^{-1} \dfrac{16210}{77915} = \tan^{-1}(0 \cdot 2036) = 11 \cdot 50°$

∴ Supply power factor angle, $\phi_S = 18{\cdot}96° + 11{\cdot}50° = 30{\cdot}46°$

∴ Supply p.f. $= \cos \phi_S = \cos 30{\cdot}46° =$ **0·86 lag**

2) *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants:*

Resistance/km/phase	= 0.1 Ω
Inductive reactance/km/phase	= 0·2 Ω
Capacitive susceptance/km/phase	= $0{\cdot}04 \times 10^{-4}$ siemen

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0·8 lagging. Use nominal T method.

Ans)

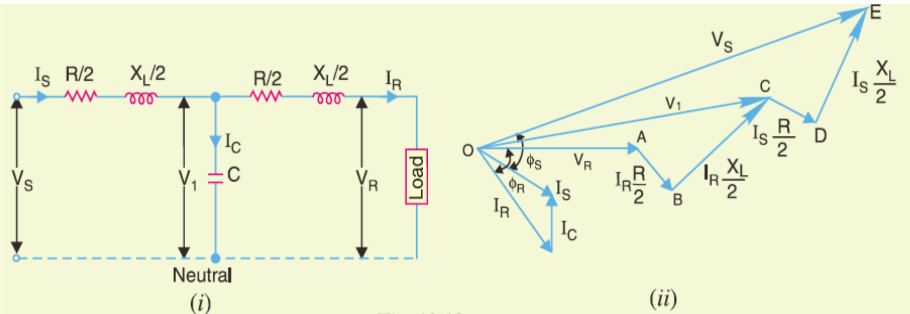


Fig. 10.13

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase. $X_L = 0{\cdot}2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0{\cdot}04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8} = 109$ A

$\cos\phi_R = 0{\cdot}8\ ;\quad \sin\phi_R = 0{\cdot}6$

Impedance per phase, $\vec{Z} = R + j\,X_L = 10 + j\,20$

(i) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\,(0{\cdot}8 - j\,0{\cdot}6) = 87{\cdot}2 - j\,65{\cdot}4$

Voltage across *C*, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 38,105 + (87{\cdot}2\ - j\,65{\cdot}4)\,(5 + j\,10)$

$= 38{,}105 + 436 + j\,872 - j\,327 + 654 = 39{,}195 + j\,545$

Charging current, $\vec{I_C} = j\,Y\,\vec{V_1} = j\,4 \times 10^{-4}(39{,}195 + j\,545) = -\,0{\cdot}218 + j\,15{\cdot}6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87{\cdot}2 - j\,65{\cdot}4)\ + (-\,0{\cdot}218 + j\,15{\cdot}6)$

$= 87{\cdot}0 - j\,49{\cdot}8 = 100\ \angle\ -29°47'$ A

∴ Sending end current $=$ **100 A**

(ii) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87{\cdot}0 - j\,49{\cdot}8)\,(5 + j\,10)$

$= 39{,}195 + j\,545 + 434{\cdot}9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\ \angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\,533$ V $=$ **69·533 kV**

(iii) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 1°40′

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = 29° 47′

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

∴ Sending end power factor, $\cos\phi_S = \cos 31°27' =$ **0·853 lag**

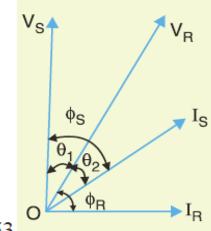


Fig. 10.14

(iv) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 40{,}145 \times 100 \times 0{\cdot}853$

$= 10273105$ W $= 10273{\cdot}105$ kW

Power delivered $= 10{,}000$ kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273{\cdot}105} \times 100 =$ **97·34%**

3) *A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is 2·5 × 10− 6 siemen/km/phase. Calculate : (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T method.*

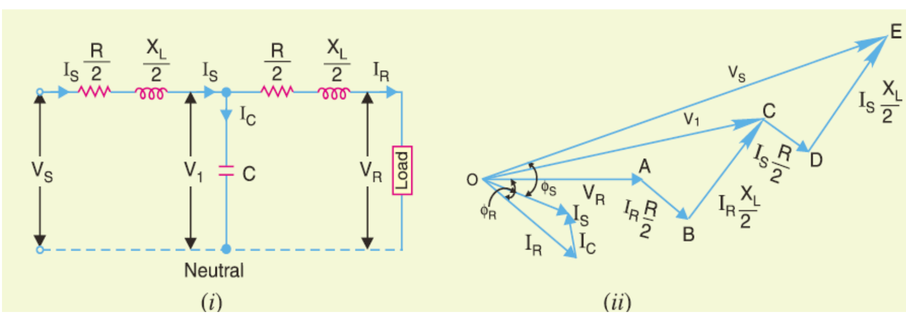
Ans)

Total resistance/phase, $R = 0{\cdot}2 \times 100 = 20\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}4 \times 100 = 40\ \Omega$

Total capacitance admittance/phase, $Y = 2{\cdot}5 \times 10^{-6} \times 100 = 2{\cdot}5 \times 10^{-4}$ S

Phase impedance, $\vec{Z} = 20 + j40$



Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508$ V

Load current, $I_R = \dfrac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 9} = 116.6$ A

$\cos \phi_R = 0 \cdot 9$; $\sin \phi_R = 0 \cdot 435$

(i) Taking receiving end voltage as the reference phasor [see phasor diagram 10.15 (*ii*)], we have,

$\vec{V_R} = V_R + j0 = 63508$ V

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R) = 116 \cdot 6 (0 \cdot 9 - j\, 0 \cdot 435) = 105 - j50 \cdot 7$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 63508 + (105 - j\, 50 \cdot 7)\,(10 + j\, 20)$

$= 63508 + (2064 + j1593) = 65572 + j1593$

Charging current, $\vec{I_C} = j\, Y\vec{V_1} = j\, 2 \cdot 5 \times 10^{-4}\, (65572 + j1593) = -0 \cdot 4 + j\, 16 \cdot 4$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (105 - j\, 50 \cdot 7) + (-0 \cdot 4 + j16 \cdot 4)$

$= (104 \cdot 6 - j\, 34 \cdot 3) = 110 \angle -18°9'$ A

∴ Sending end current $=$ **110 A**

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2$

$= (65572 + j1593) + (104 \cdot 6 - j34 \cdot 3)\,(10 + j20)$

$= 67304 + j\, 3342$

∴ Magnitude of $V_S = \sqrt{(67304)^2 + (3342)^2} = 67387\ V$

∴ Line value of sending end voltage

$= 67387 \times \sqrt{3} = 116717$ V $=$ **116·717 kV**

(ii) Total line losses for the three phases

$= 3\, I_S^{\,2}\, R/2 + 3 I_R^{\,2}\, R/2$

$= 3 \times (110)^2 \times 10 + 3 \times (116 \cdot 6)^2 \times 10$

$= 0 \cdot 770 \times 10^6$ W $= 0 \cdot 770$ MW

∴ Transmission efficiency $= \dfrac{20}{20 + 0 \cdot 770} \times 100 =$ **96·29%**

4) *A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3 × 10−6 S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.*

Ans)

Total resistance/phase, $R = 0 \cdot 1 \times 150 = 15\ \Omega$

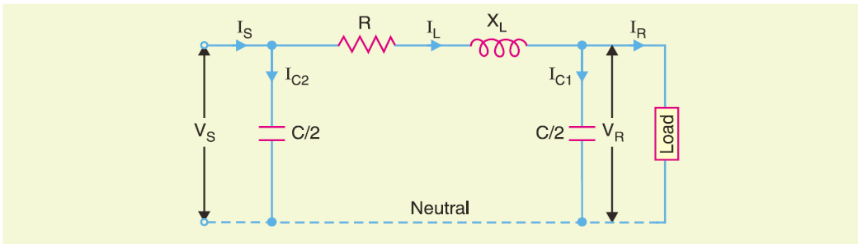
Total reactance/phase, $X_L = 0 \cdot 5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 8} = 328$ A

$\cos \phi_R = 0 \cdot 8$; $\sin \phi_R = 0 \cdot 6$



Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328\,(0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$

Charging current at the load end is

$$\vec{I_{C1}} = \vec{V_R}\,j\,\frac{Y}{2} = 63{,}508 \times j\,\frac{45 \times 10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$

$$= 63{,}508 + (262{\cdot}4 - j\,182{\cdot}5)\,(15 + j\,75)$$

$$= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687$$

$$= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881 \angle 11°47' \text{ V}$$

∴ Line to line sending end voltage = 82,881 × √3 = 1,43,550 V = **143·55 kV**

Charging current at the sending end is

$$I_{C2} = j\vec{V_S}\,Y/2 = (81{,}131 + j\,16{,}942{\cdot}5)\,j\,\frac{45 \times 10^{-5}}{2}$$

$$= -3{\cdot}81 + j\,18{\cdot}25$$

Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-3{\cdot}81 + j\,18{\cdot}25)$

$$= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4 \angle -32{\cdot}4° \text{ A}$$

∴ Sending end current = **306·4 A**

5) *A 100-km long, 3-phase, 50-Hz transmission line has following line constants:*
Resistance/phase/km = 0·1 Ω
Reactance/phase/km = 0·5 Ω
Susceptance/phase/km = 10 × 10− 6 S
If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method :
(i) sending end power factor (ii) regulation (iii) transmission efficiency

Ans)

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

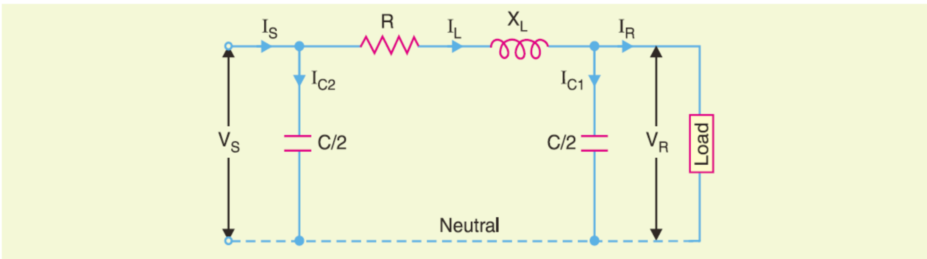
Total reactance/phase, $X_L = 0{\cdot}5 \times 100 = 50\ \Omega$

Susceptance/phase, $Y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66 \times 10^3/\sqrt{3} = 38105$ V

Load current, $I_R = \dfrac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}9} = 195$ A

$$\cos\phi_R = 0{\cdot}9 \quad ; \quad \sin\phi_R = 0{\cdot}435$$



Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 38105 \text{ V}$$

Load current, $\vec{I_R} = I_R (\cos\phi_R - j\sin\phi_R) = 195\,(0{\cdot}9 - j\,0{\cdot}435) = 176 - j\,85$

Charging current at the receiving end is

$$\vec{I_{C1}} = \vec{V_R}\, j\,\frac{Y}{2} = 38105 \times j\,\frac{10\times10^{-4}}{2} = j\,19$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (176 - j\,85) + j\,19 = 176 - j\,66$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$

$$= 38{,}105 + (176 - j\,66)\,(10 + j\,50)$$
$$= 38{,}105 + (5060 + j\,8140)$$
$$= 43{,}165 + j\,8140 = 43{,}925\,\angle 10{\cdot}65^\circ \text{ V}$$

Sending end line to line voltage $= 43{,}925 \times \sqrt{3} = 76 \times 10^3 \text{ V} = 76 \text{ kV}$

Charging current at the sending end is

$$\vec{I_{C2}} = \vec{V_S}\, jY/2 = (43{,}165 + j\,8140)\, j\,\frac{10\times10^{-4}}{2}$$
$$= -4{\cdot}0 + j\,21{\cdot}6$$

$\therefore$ Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C2}} = (176 - j\,66) + (-4{\cdot}0 + j\,21{\cdot}6)$

$$= 172 - j\,44{\cdot}4 = 177{\cdot}6\,\angle -14{\cdot}5^\circ \text{ A}$$

(*i*) Referring to phasor diagram in Fig. 10·20,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = $10{\cdot}65^\circ$

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = $-14{\cdot}5^\circ$

$\therefore$ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$ = $\theta_2 + \theta_1$

$= 14{\cdot}5^\circ + 10{\cdot}65^\circ = 25{\cdot}15^\circ$

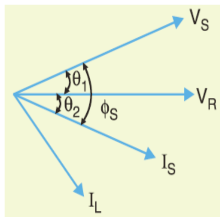


Fig. 10.20

$\therefore$ Sending end p.f., $\cos\phi_S = \cos 25{\cdot}15^\circ =$ **0·905 lag**

(*ii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{43925 - 38105}{38105} \times 100 =$ **15·27 %**

(*iii*) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 43925 \times 177{\cdot}6 \times 0{\cdot}905$

$= 21{\cdot}18 \times 10^6 \text{ W} = 21{\cdot}18 \text{ MW}$

Transmission efficiency $= (20/21{\cdot}18) \times 100 =$ **94 %**