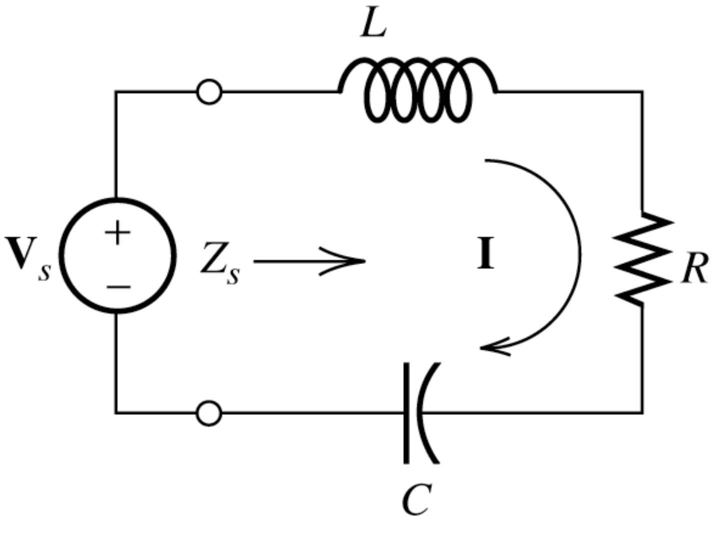
ELG4139: Passive Filters

- A filter is a system that processes a signal in some desired fashion.
- There are two broad categories of filters:
  - An **analog filter** processes continuous-time signals
  - A **digital filter** processes discrete-time signals.
- The analog or digital filters can be subdivided into four categories:
  - Lowpass Filters
  - Highpass Filters
  - Bandstop Filters
  - Bandpass Filters

Series Resonance



$$Z_s(f) = j2\pi fL + R - j\frac{1}{2\pi fC}$$

For resonance :

$$2\pi f_0 L = \frac{1}{2\pi f_0 C} \rightarrow f_0^2 = \frac{1}{(2\pi)^2 LC}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_s \equiv \frac{Reactance\ \ of\ \ inductance\ at\ resonance}{Resistance}$$

$$= \frac{2\pi f_0 L}{R}$$

$$Substitute\ L = \frac{1}{(2\pi)^2 (f_0)^2 C} \quad from\ \ f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_s = \frac{1}{2\pi f_0 CR}$$

Series Resonance

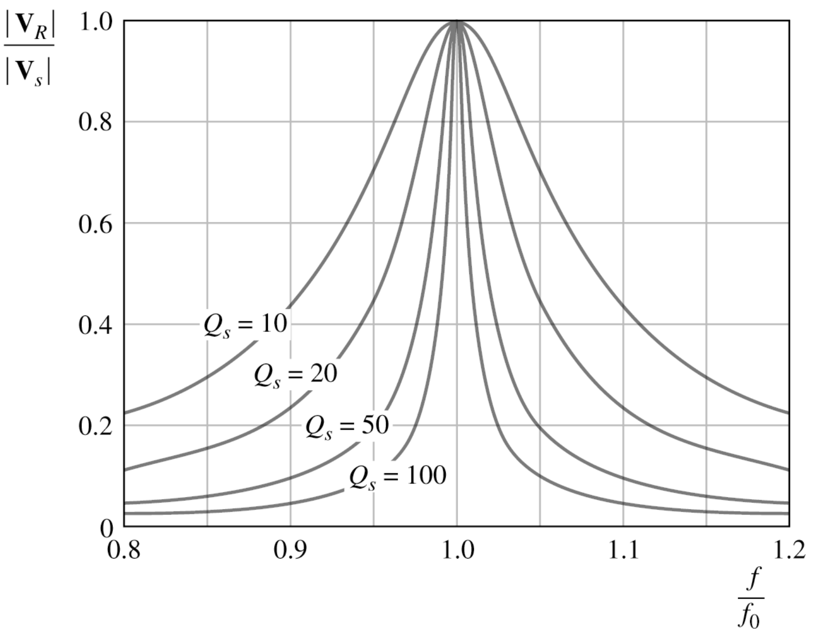
$$Z_s(f) = R + j2\pi fL - j\frac{1}{2\pi fC}$$

$$= R\left[1 + j\left(\frac{2\pi fL}{R} - \frac{1}{2\pi fRC}\right)\right]$$

$$= R\left[1 + j\frac{2\pi f_0 L}{R}\left(\frac{f}{f_0} - \frac{1}{(2\pi)^2 ff_0 LC}\right)\right]$$

$$Substitute\ f_0 = \frac{1}{2\pi\sqrt{LC}}\ and\ Q_s = \frac{2\pi f_0 L}{R}$$

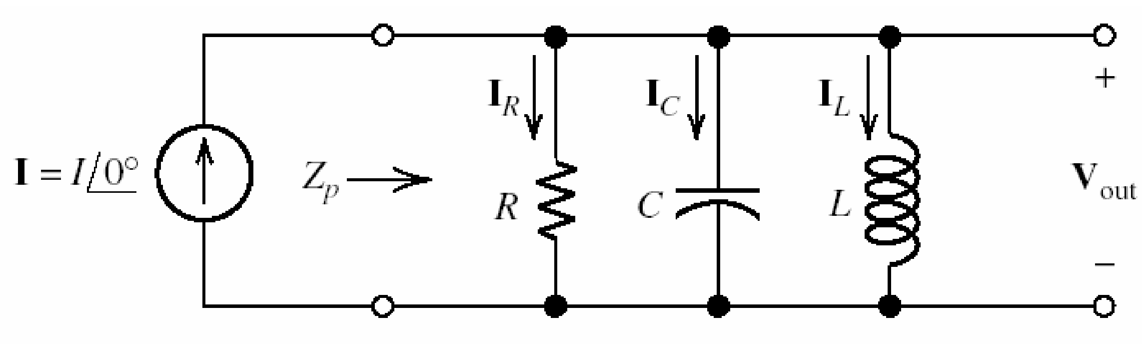
$$Z_s(f) = R\left[1 + jQ_s\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right]$$

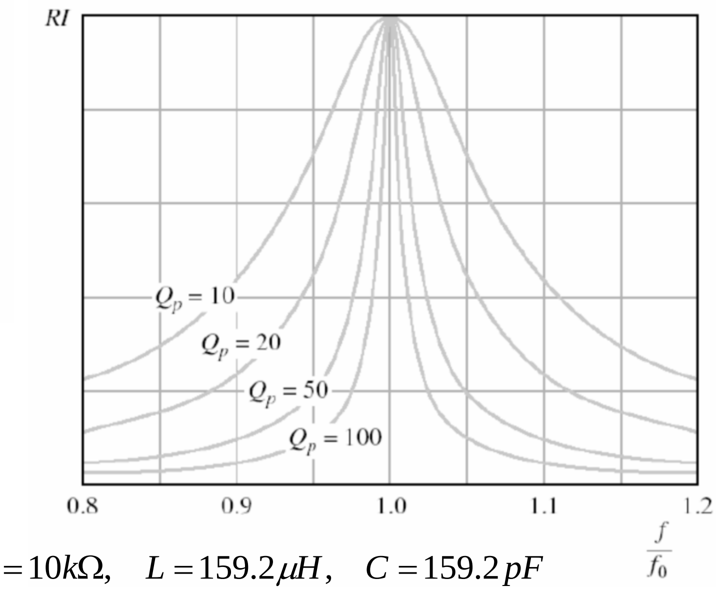


$$\mathbf{I} = \frac{\mathbf{V}_s}{Z_s(f)} = \frac{\mathbf{V}_s / R}{1 + jQ_s\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

$$\mathbf{V}_R = R\mathbf{I} = \frac{\mathbf{V}_s}{1 + jQ_s\left(\frac{f}{f_0} - \frac{f_0}{f}\right)} \rightarrow \frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{1}{1 + jQ_s\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

Parallel Resonance





$$Z_p = \frac{1}{(1/R) + j2\pi fC - j(1/2\pi fL)}$$

At resonance Z_P is purely resistive:

$$j2\pi f_0 C = j(1/2\pi f_0 L) \rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_P \equiv \left(\frac{Resistance}{Reactance\ of\ inductance\ at\ resonance}\right)$$

$$= \frac{R}{2\pi f_0 L}$$

$$Substitute\ L = \frac{1}{(2\pi)^2 (f_0)^2 C} \quad from\ f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_P = 2\pi f_0 CR$$

$$\mathbf{I} = 10^{-3}\angle 0°, \quad R = 10k\Omega, \quad L = 159.2\mu H, \quad C = 159.2\,pF$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1x10^6\,Hz \quad Q_P = \frac{R}{2\pi f_0 L} = \frac{10^4}{2\pi(1x10^6\,Hz)(159.2\mu H)} = 10$$

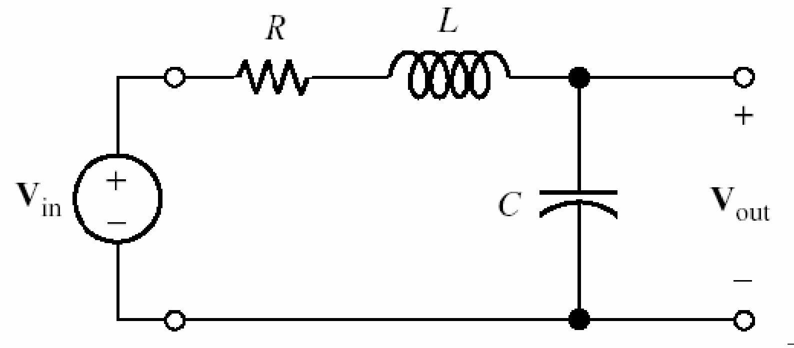
$$\mathbf{V}_{out} = \mathbf{I}R = 10^{-3}\angle 0°(10^4) = 10\angle 0°$$

$$\mathbf{I}_R = \frac{\mathbf{V}_{out}}{R} = \frac{\mathbf{V}_{out}}{10^4} = \frac{10\angle 0°}{10^4} = 10^{-3}\angle 0°$$

$$\mathbf{I}_L = \frac{\mathbf{V}_{out}}{Z_L} = \frac{\mathbf{V}_{out}}{j2\pi f_0 L} = \frac{10\angle 0°}{j10^3} = 10^{-2}\angle -90°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{out}}{Z_C} = \frac{\mathbf{V}_{out}}{\frac{-j}{2\pi f_0 C}} = \frac{10\angle 0°}{-j10^3} = 10^{-2}\angle 90°$$

Second Order Low-Pass Filter



$$\mathbf{V}_{out} = \frac{Z_C}{Z_R + Z_L + Z_C}\mathbf{V}_{in} = \frac{\frac{-j}{2\pi fC}}{R + j2\pi fL - \frac{j}{2\pi fC}}\mathbf{V}_{in} = \frac{\frac{-j}{2\pi fRC}}{1 + j\frac{2\pi f_0 L}{R}\left(\frac{f}{f_0} - \frac{1}{2\pi f f_0 LC}\right)}\mathbf{V}_{in}$$

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = H(f) = \frac{\frac{-j}{2\pi fRC}}{1 + j\frac{2\pi f_0 L}{R}\left(\frac{f}{f_0} - \frac{1}{2\pi f f_0 LC}\right)} = \frac{-jQ_S(f/f_0)}{1 + jQ_S\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{-jQ_s(f_0/f)}{1 + jQ_s(f/f_0 - f_0/f)}$$

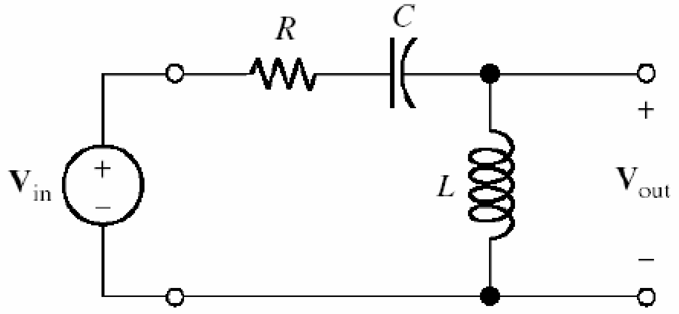
$$= \frac{Q_s(f_0/f)\angle -90^\circ}{\sqrt{1 + Q_S^2(f/f_0 - f_0/f)^2}\angle Tan^{-1}Q_s(f/f_0 - f_0/f)}$$

$$|H(f)| = \frac{Q_s(f_0/f)}{\sqrt{1 + Q_S^2(f/f_0 - f_0/f)^2}}$$

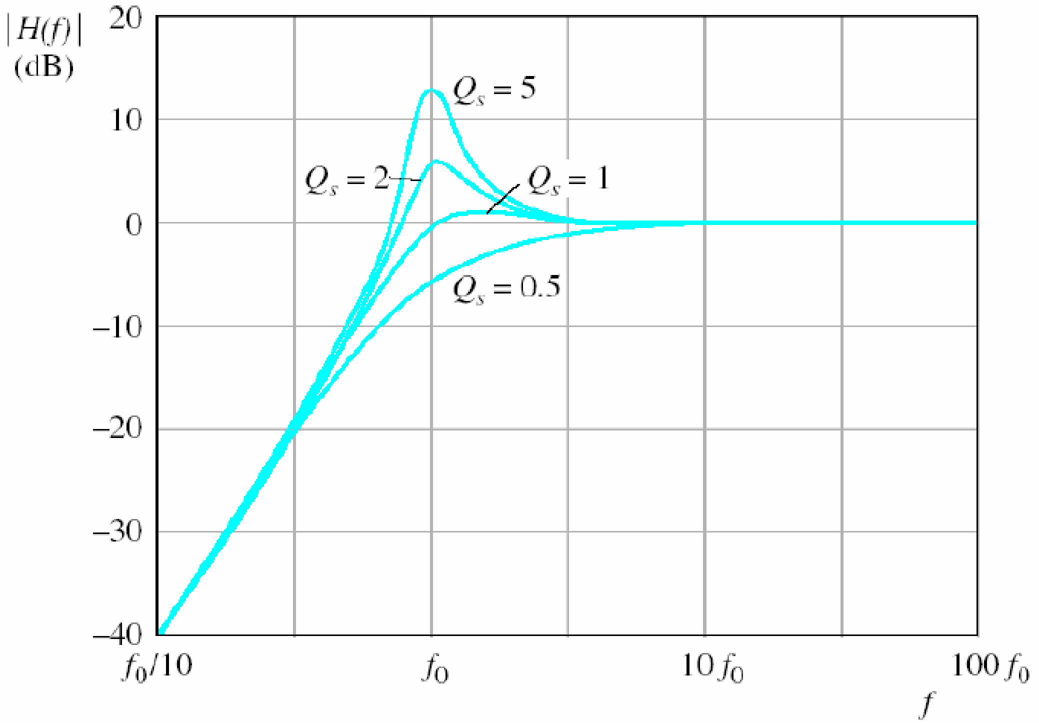
Second Order High-Pass Filter

At low frequency the capacitor is an open circuit

At high frequency the capacitor is a short and the inductor is open



(a) Circuit diagram

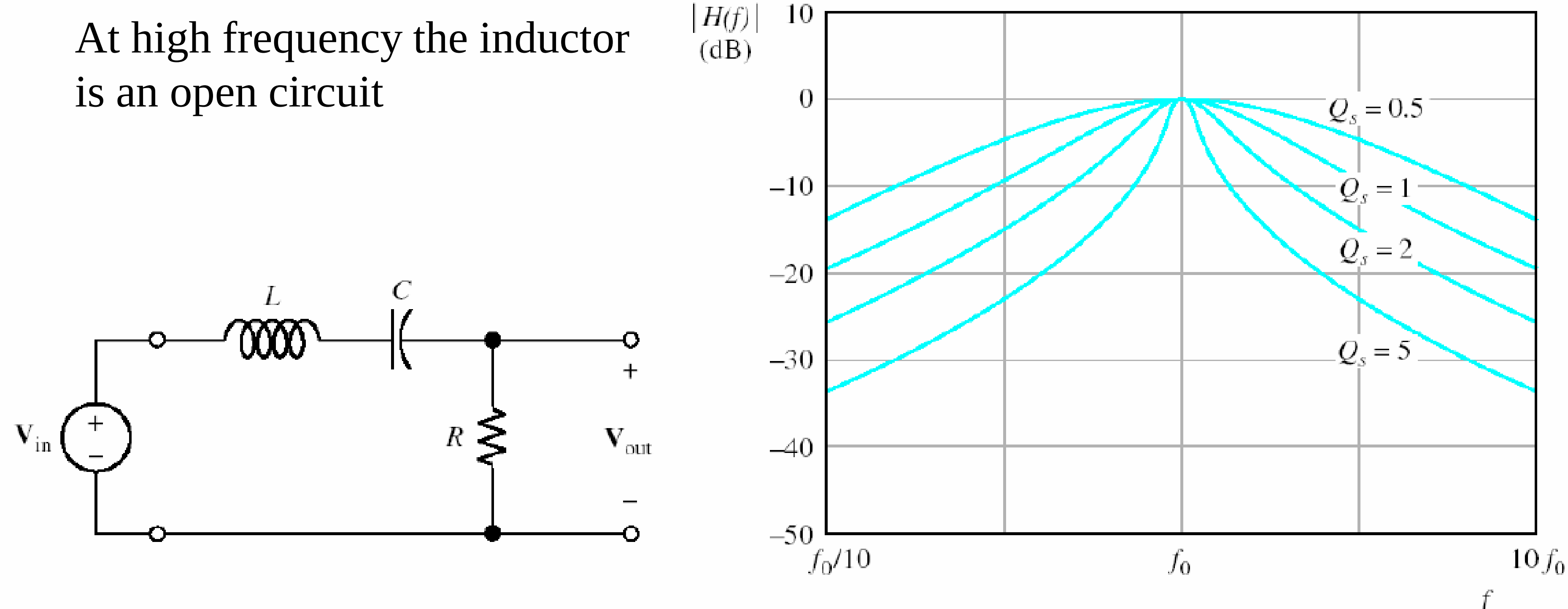


(b) Transfer-function magnitude

Second Order Band-Pass Filter

At low frequency the capacitor is an open circuit

At high frequency the inductor is an open circuit



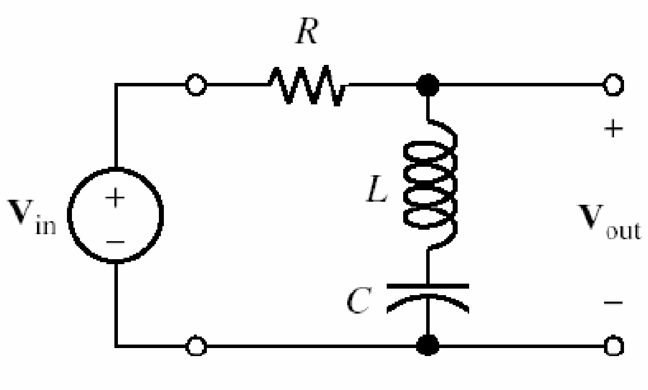
(a) Circuit diagram

(b) Transfer-function magnitude

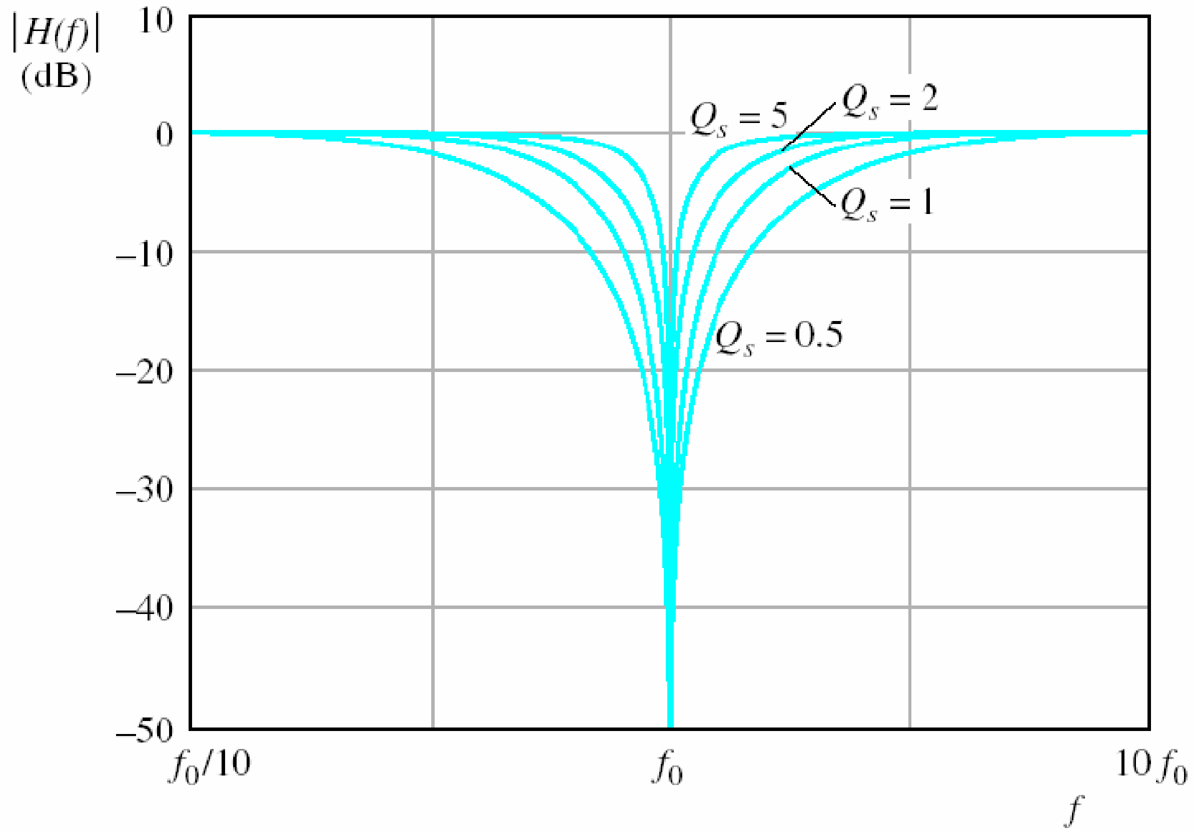
Second Order Band-Reject Filter

At low frequency the capacitor is an open circuit

At high frequency the inductor is an open circuit

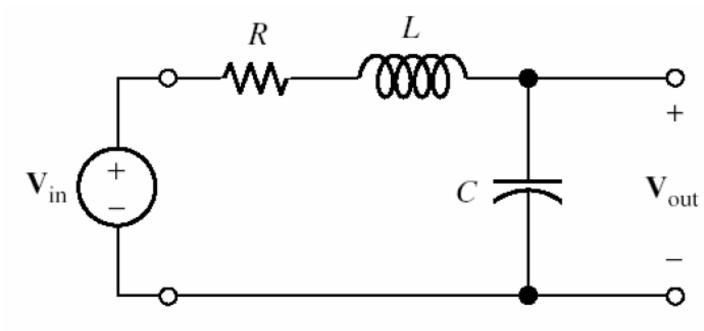


(a) Circuit diagram

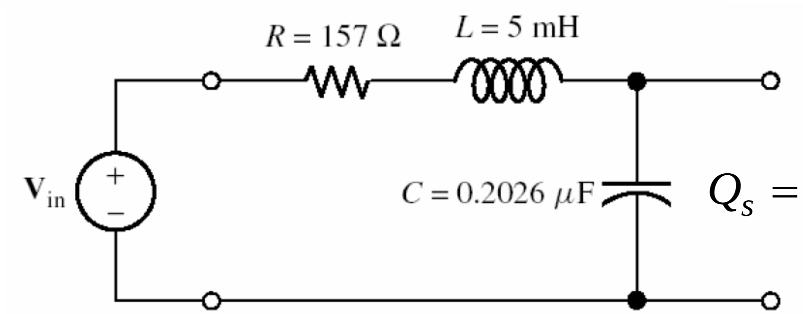


(b) Transfer-function magnitude

Example: Design a filter with $Q_S = 1$ that passes frequency components lower than 5 kHz and rejects components higher than 5 kHz. Chose L=5 mH.

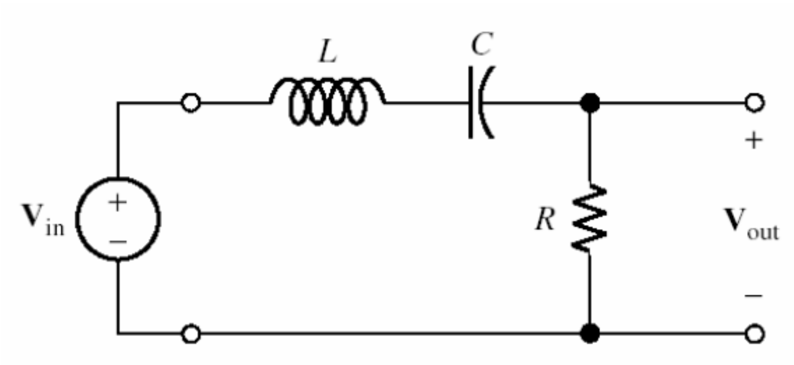


$$f_0 = 5kHz = \frac{1}{2\pi\sqrt{LC}} \rightarrow C = \frac{1}{(2\pi)^2 f_0^2 L} = \frac{1}{(2\pi)^2 (5x10^3)^2 (5x10^{-3})} = 0.2026\mu F$$



$$Q_s = \frac{2\pi f_0 L}{R} \rightarrow R = \frac{2\pi f_0 L}{Q} = \frac{2\pi(5kHz)(5x10^{-3}H)}{1} = 157.1\Omega$$

Example: Design a filter that passes frequency components between f_L=45 kHz and f_H=55 kHz. Choose L=1 mH.



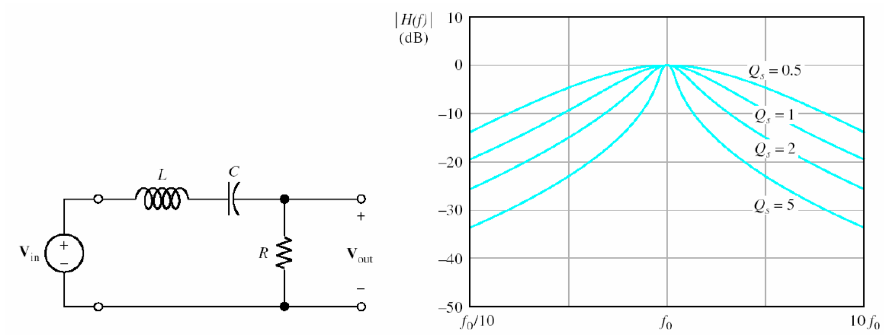
$$f_0 = 50kHz = \frac{1}{2\pi\sqrt{LC}} \rightarrow C = \frac{1}{(2\pi)^2 f_0^2 L} = \frac{1}{(2\pi)^2 (50x10^3)^2 (1x10^{-3})} = 10.13nF$$

$$B = f_H - f_L = 10kHz$$

$$Q = \frac{f_0}{B} = \frac{50kHz}{10kHz} = 5$$

$$R = \frac{2\pi f_0 L}{Q}$$

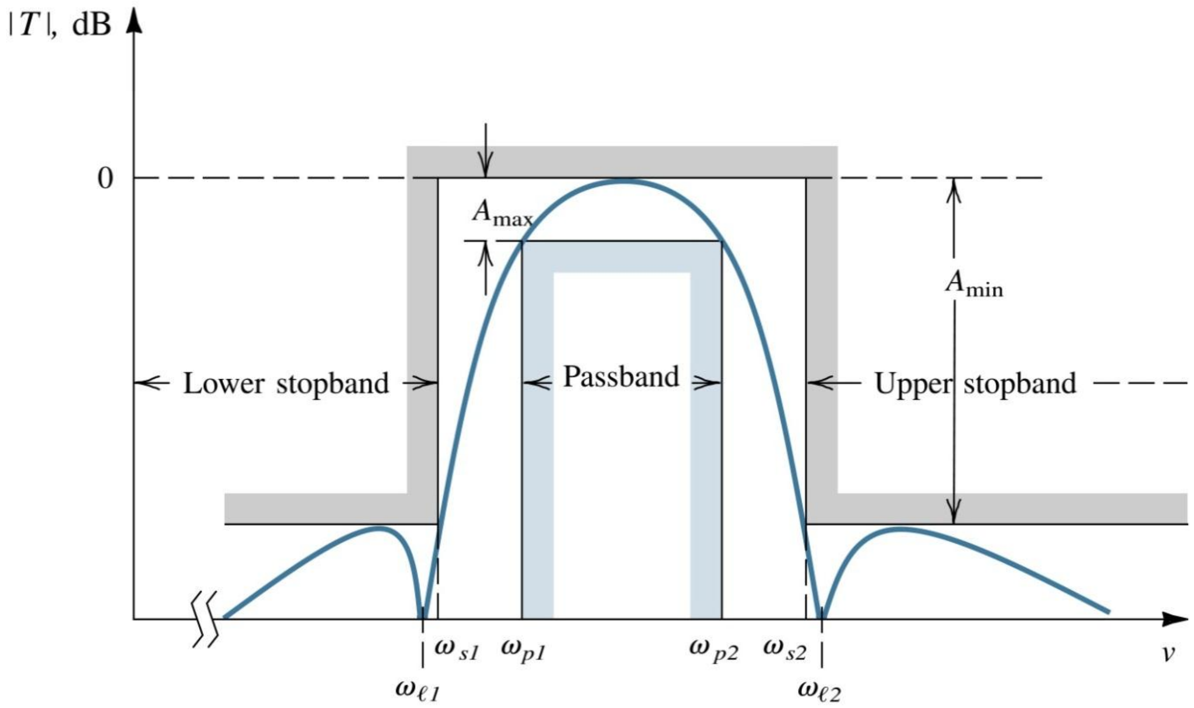
$$= \frac{2\pi(50kHz)(1x10^{-3}H)}{5} = 62.83\Omega$$



(a) Circuit diagram

(b) Transfer-function magnitude

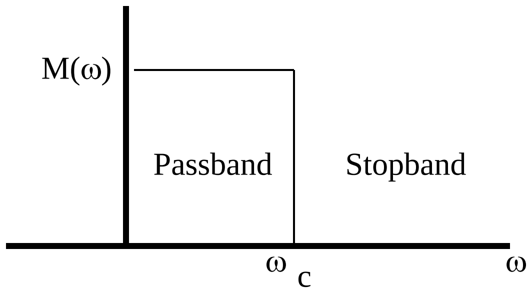
Filter Specification



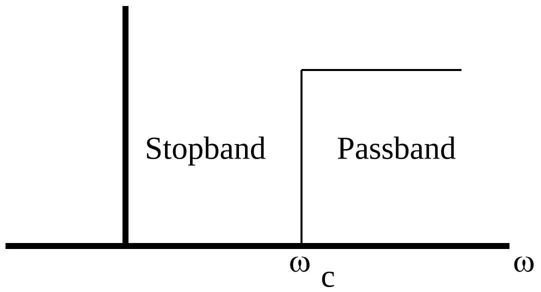
Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

Ideal Filters

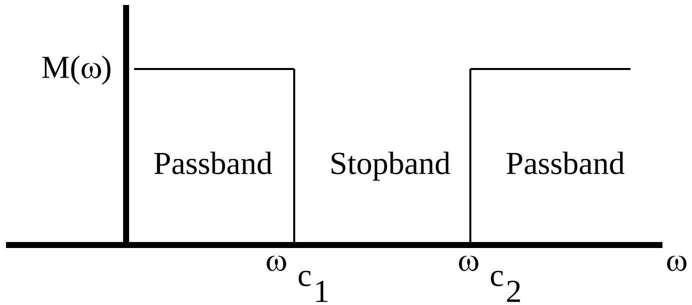
Lowpass Filter



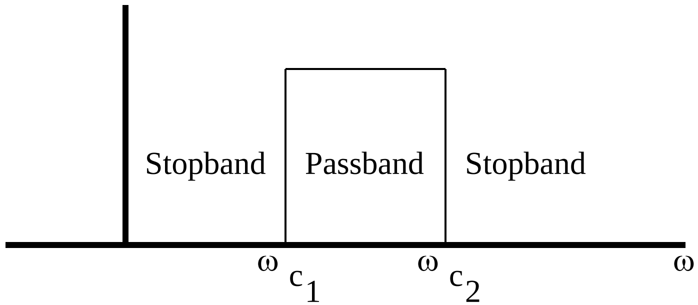
Highpass Filter



Bandstop Filter



Bandpass Filter



Passive and Active Filters

- **Passive filters** employ resistors, capacitors, and inductors (RLC networks). To minimize distortion in the filter characteristic, it is desirable to use inductors with high quality factors, however these are difficult to implement at frequencies below 1 kHz, because.
  - They are non-ideal (lossy)!
  - They are bulky and expensive!
- **Active filters** overcome these drawbacks and are realized using resistors, capacitors, and active devices (usually op-amps) which can all be integrated:
  - Active filters replace inductors using op-amp based equivalent circuits.

- **The Problem of the Inductor!**
- High accuracy (1% or 2%), small physical size, or large inductance values are required!
- Standard values of inductors are not very closely spaced.
- Difficult to find an off-the-shelf inductor within 10 percent of any arbitrary value.
- Adjustable inductors are used.
- Tuning such inductors to the required values is time-consuming and expensive for larger quantities of filters.
- Inductors are often prohibitively expensive.

Op Amp-based Active Filters

- **Advantages:**
  - Reduced size and weight, and therefore parasitics.
  - Increased reliability and improved performance.
  - Simpler design than for passive filters and can realize a wider range of functions as well as providing voltage gain.
  - In large quantities, the cost of an IC is less than its passive counterpart.
- **Disadvantages:**
  - Limited bandwidth of active devices limits the highest attainable pole frequency and therefore applications above 100 kHz (passive RLC filters can be used up to 500 MHz).
  - The achievable quality factor is also limited.
  - Require power supplies (unlike passive filters).
  - Increased sensitivity to variations in circuit parameters caused by environmental changes compared to passive filters.
- For applications, particularly in voice and data communications, the economic and performance advantages of active RC filters far outweigh their disadvantages.

Bode Plots and dB

- Bode plots are important when considering the frequency response characteristics of amplifiers. They plot the magnitude or phase of a transfer function in dB versus frequency.
- Two levels of power are compared using a unit of measure called the ***bel***.

$$B = \log_{10} \frac{P_2}{P_1}$$

The ***decibel*** is defined as:

1 bel = 10 decibels (dB)

$$dB = 10 \log_{10} \frac{P_2}{P_1}$$

A common dB term is the **half power point** which is the dB value when the P_2 is one- half P_1.

$$10 \log_{10} \frac{1}{2} = -3.01 dB \approx -3 dB$$

Logarithms

- A logarithm is a linear transformation used to simplify mathematical and graphical operations. It is a one-to-one correspondence.
- Any number *(N)* can be represented as a *base number (b)* raised to a *power (x)*.

$$N = (b)^x$$

- The value *power (x)* can be determined by taking the logarithm of the *number (N)* to *base (b)*.

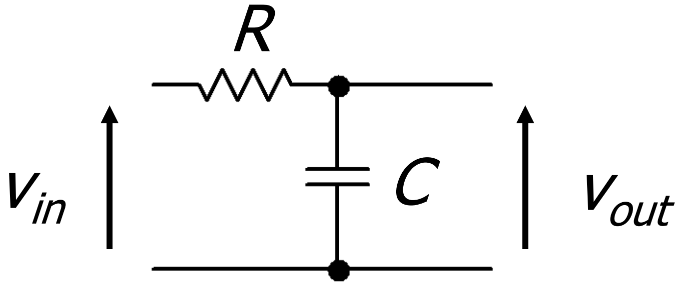
$$x = \log_b N$$

- Although there is no limitation on the numerical value of the base, calculators are designed to handle either base 10 (the common logarithm) or base e (the natural logarithm). Any base can be found in terms of the common logarithm by:

$$\log_q w = \frac{1}{\log_{10} q} \log_{10} w$$

Poles and Zeros of the Transfer Function

- Pole—value of *s* where the denominator goes to zero.
- Zero—value of *s* where the numerator goes to zero.
- Single-Pole Passive Filters (First-Order Low-Pass Filter)



$$\frac{v_{out}}{v_{in}} = \frac{Z_C}{R + Z_C} = \frac{1/sC}{R + 1/sC}$$

$$= \frac{1}{sCR + 1} = \frac{1/RC}{s + 1/RC}$$

- Cut-off frequency = 1/*RC* rad/s
- **Disadvantage:** Any load (or source) impedance will change the frequency response.