

# Model and Control for Supercapacitor-based Energy Storage System for Metro Vehicles

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**Abstract**—In order to improve the simulation efficiency, this paper presents a kind of switch-averaged model for the energy storage system to analyze its stability. This energy storage system is applied in metro vehicles that use supercapacitors as the energy storage elements to provide desired management of the power flows. In order to achieve quick response, an effective and simple control method using state feedback is presented. The effectiveness of the proposed control method is verified by simulation, and the simulation efficiency of the switch-averaged model is proved by compared with the macro model. The modeling method and control method here can be compatible with other DC/DC converters

**Keywords**—Switch-averaged Model; State Feedback; Energy Storage System; Bi-directional DC/DC Converter

## I. INTRODUCTION

Increasing efficiency in the conversion, delivery and utilization of energy is an essential part of a comprehensive national energy policy. Specifically, it can help facilitate energy efficiency by developing energy storage system technologies, the primary purpose of which is to store energy during off-peak hours and to be used during peak hours, to further reduce energy losses in electric power generation, transmission and distribution.

Now, most energy in heavy vehicles, such as trucks, buses, and particularly metro vehicles, is wasted during braking. Supercapacitor-based energy storage system is a very good way to recover braking energy for high power (more than 4Mw) and high voltage (DC 1500V) metro vehicles [1]. Fig 1 (a) shows the schematic diagram of the energy storage system and metro power system. When the metro vehicle is braking, the mechanical energy of rotation is converted to the electrical power by using an electromagnetic brake. Now, the regenerative power is stored in supercapacitor-based energy storage system instead of being consumed by the braking resistor, if there is no other vehicle accelerating.

Here, the energy storage system consists of the charge and discharge unit and supercapacitor bank. Fig 1 (b) shows the topology of the charge and discharge unit which is a non-isolated bi-directional DC/DC converter actually [2]. To be noted, the input filter ( $L_{filter}$  and  $C_1$ ) is used to smooth the input

pulse current.

With help of energy storage system, metro vehicles could realize quick and effective recuperation of energy in frequent braking and deceleration in city. However, it's a very challenging task to control a non-isolated bi-directional DC/DC converter because of the wide voltage range and high current of supercapacitor bank.

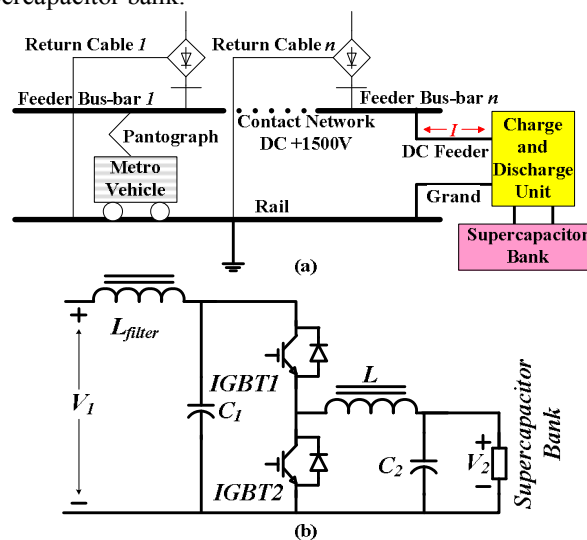


Fig. 1. The topology of the energy storage system

This paper presents a high-efficient model of the energy storage system and an effective control method.

## II. MODEL OF THE ENERGY STORAGE SYSTEM

Generally, switch-averaged model is used to predict converter steady-state characteristics, small signal or large signal dynamics, which can be obtained by separating the switching elements from the remainder of the converter, and replacing the switch network by the model. But it is difficult to used for analyze multiple switch converter and it doesn't do help to the system design. So this paper uses switch-averaged model to analyze the non-isolated bi-directional DC/DC converter, and utilizes a simply state-averaged model, which is a mathematical model, to design the control method for the converter.

The basic assumptions are made that:

1. The natural time constant of the converter are much longer than the switching period, so that the converter

- contains low-pass filtering of the switching harmonics.
- 2. IGBT switches and the diodes have the same ON resistance  $r(\Omega)$  approximately, which could simplify the model analysis.
- 3. Neglecting the effect of the input filter ( $L_{filter}$  and  $C_1$ ).

These IGBT switches are complementary ones, IGBT1 conducts for duty  $D$  of each switching cycle and IGBT2 conducts for the remainder  $1-D$ , as shown in Fig. 2.

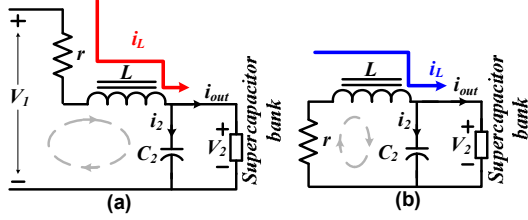


Fig. 2. The states of the converter

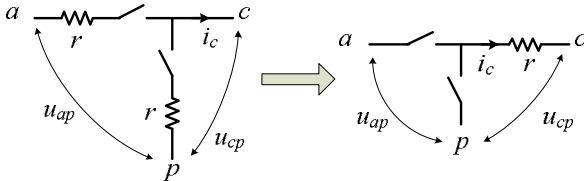


Fig. 3. The switch-averaged model

The work mode of the bi-directional DC/DC converter is very similar to Buck converter in continuous current mode [3]. When the IGBT1 switch is on and IGBT2 switch is off, the inductor current flows through IGBT1, and its current and voltage satisfy the equation below:

$$\begin{aligned} i_a &= i_c \\ V_{cp} &= V_{ap} - ri_a \end{aligned} \quad (1)$$

When the IGBT1 switch is off and IGBT2 switch is on, the inductor current flows through IGBT2, and its current and voltage satisfy the equation below:

$$\begin{aligned} i_a &= 0 \\ V_{cp} &= -ri_c \end{aligned} \quad (2)$$

Here, using (1) and (2), the averaging process can be achieved directly on the switching converter itself:

$$\begin{aligned} i_a &= Di_c \\ V_{cp} &= DV_{ap} - ri_c \end{aligned} \quad (3)$$

From (3), we can conclude that two switches work as an ideal DC transformer, and the resistant  $r$  can be transformed from branch  $a$  to branch  $c$ , keeping the loss energy equivalent, as shown in Fig. 3.

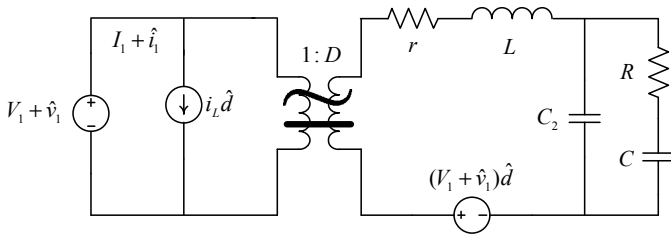


Fig. 4. The large signal equivalent circuit model of the energy storage system Using (3), we can obtain large signal equivalent circuit model, as shown in Fig. 4, which is very helpful to verify the

performance of the model designed because of its high simulation efficiency. In order to realize the ideal DC transformer, the switch IGBT1 can be modeled by a controlled current source, and the IGBT2 can be modeled by a controlled voltage source. Noted, the  $\hat{d}$  here is the control disturbance.

### III. CONTROL OF THE ENERGY STORAGE SYSTEM

Because the regeneration energy of the metro vehicles is very large while the capacity of the energy storage system prototype is relatively small (200kW), we use constant current control here.

When IGBT1 turns on and IGBT2 turns off, a circuit equation of the circuit model Fig. 2(a) is given by [4]:

$$ri_L + L \frac{di_L}{dt} + V_2 - V_1 = 0 \quad (4)$$

$$i_L = C_2 \frac{dV_2}{dt} + i_{out} \quad (5)$$

When IGBT1 turns off and IGBT2 turns on, a circuit equation of the circuit model Fig. 2(b) is given by:

$$ri_L + L \frac{di_L}{dt} + V_2 = 0 \quad (6)$$

$$i_L = C_2 \frac{dV_2}{dt} + i_{out} \quad (7)$$

Using the state-space averaging approach, we can obtain:

$$\frac{dx}{dt} = Ax + bDV_1 + d \quad (8)$$

$$\text{Here, } x = \begin{bmatrix} i_L \\ V_2 \end{bmatrix}, A = \begin{bmatrix} -r/L & -1/L \\ 1/C_2 & 0 \end{bmatrix}, b = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}, d = \begin{bmatrix} 0 \\ -i_{out}/C_2 \end{bmatrix}$$

The state-averaged model (8) can be treated as a linear time-invariant system with a control input  $D$  and an external input  $d$ , because  $V_1$  can be measured accurately by a voltage sensor.

An ideal reference state of the system is defined as:

$$x^* = \begin{bmatrix} i_L^* \\ V_2 \end{bmatrix}, x_e = x - x^* \quad (9)$$

Where  $i_L^*$  is constant and  $V_2$  is time-varying. And we can obtain the ideal reference equation:

$$\frac{dx^*}{dt} = Ax^* + bD^*V_1^* + d \quad (10)$$

Subtract (10) from (8), then:

$$\frac{dx_e}{dt} = Ax_e + bDV_1 - bD^*V_1^* \quad (11)$$

The control input is:

$$D = \frac{1}{V_1} (-K_a x_e + D^*V_1^*), K_a = [K_i \quad K_r] \quad (12)$$

And, the closed-loop system can be given:

$$\frac{dx_e}{dt} = (A - bK_a)x_e \quad (13)$$

When feedback gain  $K_a$  is chosen in order to make matrix  $A - bK_a$  stable, the error vector  $x_e$  approaches 0. And the bilinear term  $D^*V_1^*$  in Equation (12) can be obtain by (10):

$$D^*V_1^* = L \frac{di_L^*}{dt} + ri_L^* + V_2 \quad (14)$$

Using (12) and (14), the energy storage system can be controlled effectively.

#### IV. SIMULATION AND RESULTS

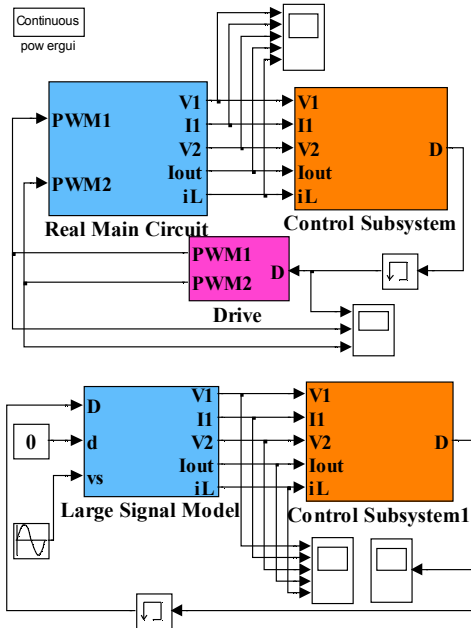


Fig. 5. The macro model and large signal model

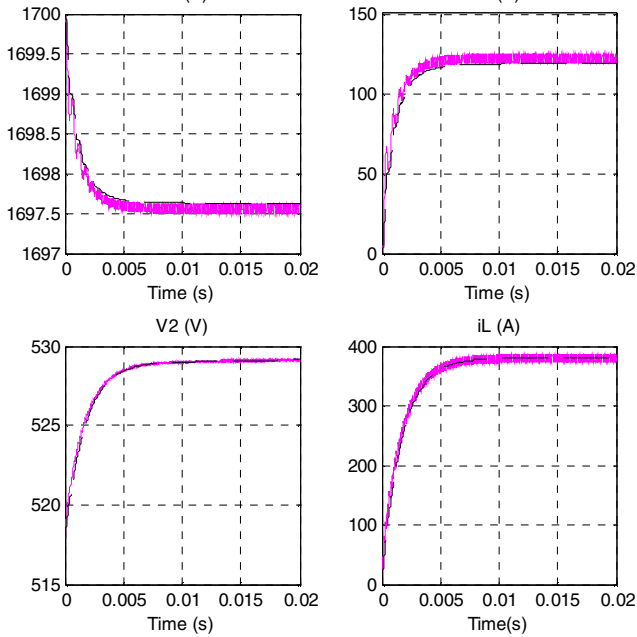


Fig. 6. The simulation results

In order to verify the effective of the control method, the model of the energy storage system for Simulink is built, as shown in Fig. 5. And the model parameters are given below:

$$V_1 = 1700V, L_{filter} = 10\mu H, L = 1.7mH, C_1 = C_2 = 800\mu F, \quad (15)$$

$$r = 0.001\Omega, V_2 = 518 \sim 1036V, R = 0.0288\Omega, C = 36F$$

The simulation results are shown in Fig. 6, the magenta solid lines are given by the macro model and the black dashed lines are given by the large signal model. We can conclude that the large signal model is as effective as the macro model, meanwhile, its simulation efficiency is much higher than the macro model, the former one takes only about 0.2s to finish the task, by contrast, the later one needs about 2.5s.

Fig. 6 shows that the control method can keep the  $i_L$  constant in the steady state, and when we change the initial states of  $i_L$  and  $V_2$ , the control method can still work very well, as shown in Fig. 7. The effectiveness of the control method presented here is verified by simulation.

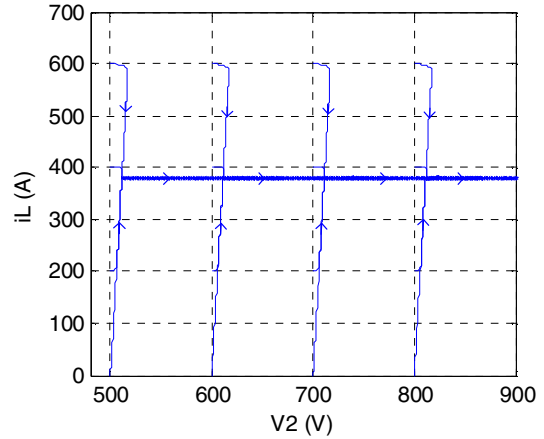


Fig. 7. Phase-plane diagram

#### V. CONCLUSION

This paper presents a high-efficient switch-averaged model and a simply effective control method of a regenerative power storage system composed by a supercapacitor-based non-isolated bi-directional DC/DC converter to save the braking energy of the metro vehicles. The control system designed here can achieve the stability and the quick response, and the efficiency of the model and effectiveness of the proposed control method are verified by simulation.

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